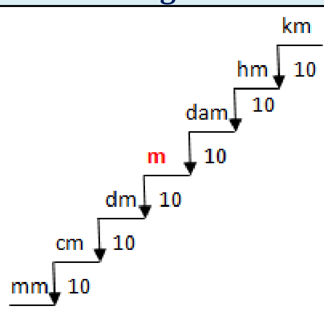
Geometrie – pentru pregătirea Evaluării Naționale la Matematică

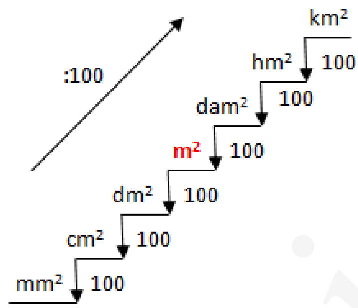
(Cls. a V a , a VI a, a VII a)

UNITĂȚI DE MĂSURĂ

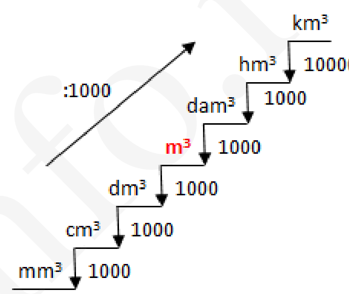
Lungime



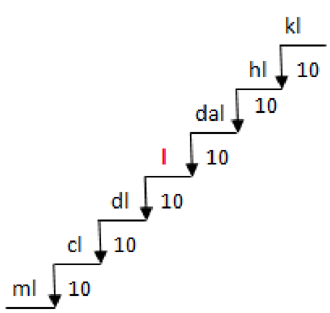
Arie



Volum



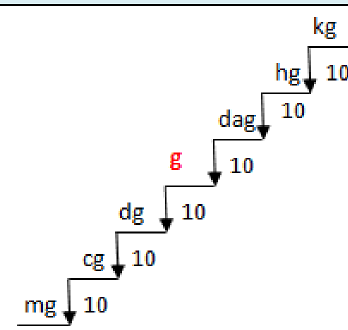
Capacitate



DE REȚINUT !

$$1hm^2 = 1ha$$
$$1dam^2 = 1ar$$
$$1dm^3 = 1l$$
$$1q = 100kg$$
$$1t = 1000kg$$
$$1v = 10000kg$$

Masă



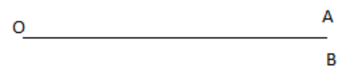
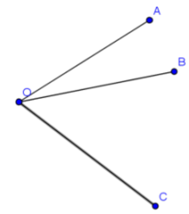
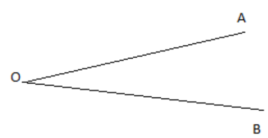
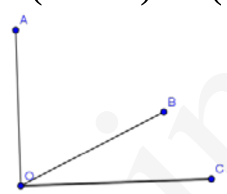
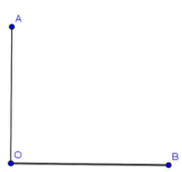
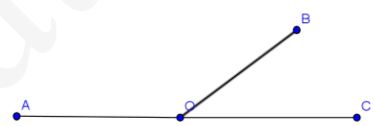
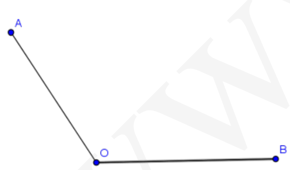
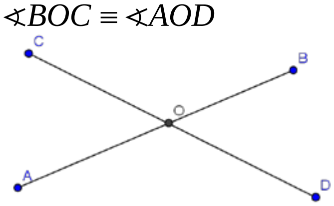
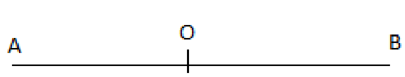
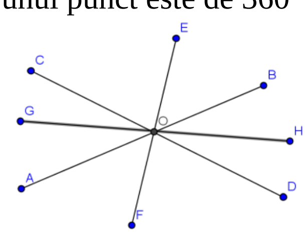
Timp - secundă, minut, ora, ziua, saptamana, luna, anul, deceniul, secol (veac), mileniu

1 deceniu = 10 ani ; 1 secol = 100 ani ; 1 mileniu = 1000 ani

Unghi - gradul, minutul, secunda

$$1^0 = 60', 1' = 60'', 1^0 = 3600''$$

UNGHIUL - Tipuri de unghiuri

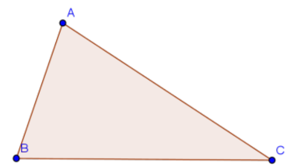
Unghi nul $m(\sphericalangle AOB)=0^0$.  	**Unghiuri adiacente** $m(\sphericalangle AOC)=m(\sphericalangle AOB)+m(\sphericalangle BOC)$ 
Unghi ascuțit $0^0<m(\sphericalangle AOB)<90^0$  	**Unghiuri complementare** $m(\sphericalangle AOB)+m(\sphericalangle BOC)=90^0$ 
Unghi drept $m(\sphericalangle AOB)=90^0$  	**Unghiuri suplementare** $m(\sphericalangle AOB)+m(\sphericalangle BOC)=180^0$ 
Unghi obtuz $90^0<m(\sphericalangle AOB)<180^0$  	**Unghiuri opuse la vârf** $\sphericalangle AOC\equiv\sphericalangle BOD$ $\sphericalangle BOC\equiv\sphericalangle AOD$ 
Unghi alungit $m(\sphericalangle AOB)=180^0$  	**Unghiuri în jurul unui punct** Suma măsurilor unghiurilor formate în jurul unui punct este de 360^0 

TRIUNGHIUL

1. Clasificare:

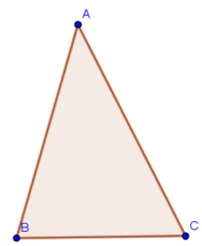
După laturi

Triunghi scalen (oarecare) = triunghiul cu laturile de lungimi diferite



Triunghi isoscel = triunghiul care are două laturi congruente

$[AB] \equiv [AC]$

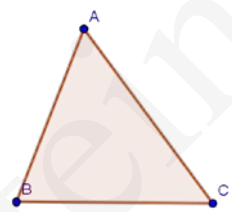


Proprietati:

1) $\sphericalangle B \equiv \sphericalangle C$
2) [AD] bisectoarea unghiului de la varf $\Rightarrow$ [AD] mediană, înălțimea și mediatoarea bazei

Triunghi echilateral = triunghiul care are toate laturile congruente

$[AB] \equiv [AC] \equiv [BC]$



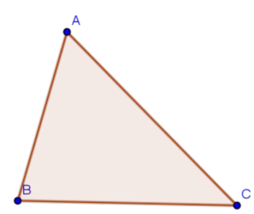
Proprietăți:

1) $m(\sphericalangle A) = m(\sphericalangle B) = m(\sphericalangle C) = 60^0$

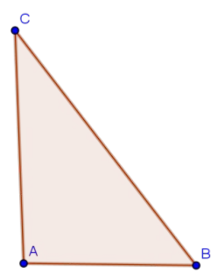
2)Bisectoarea oricărui unghi este mediană, înălțime și mediatoare

Dupa unghiuri

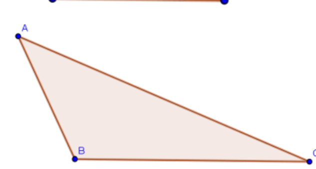
Triunghi ascuțitunghic = triunghiul care are toate unghiurile ascuțite (<90)



Triunghi dreptunghic = triunghiul care are un unghi drept (90^0)



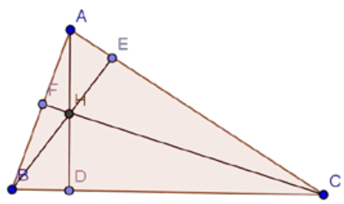
Triunghi obtuzunghic = triunghiul care un unghi obtuz ($>90^0$)

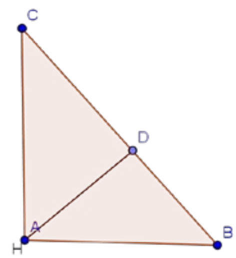


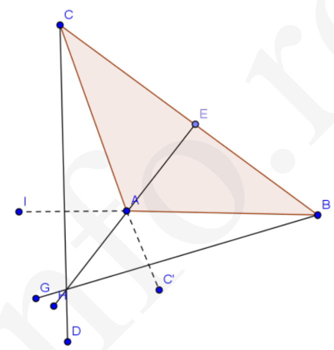
2. Linii importante in triunghi

a) **Înălțimea** ***= segmentul determinat de un vârf al triunghiului și proiecția acestuia pe latura opusă***

- Intersecția înălțimilor este *ortocentrul* triunghiului (H)

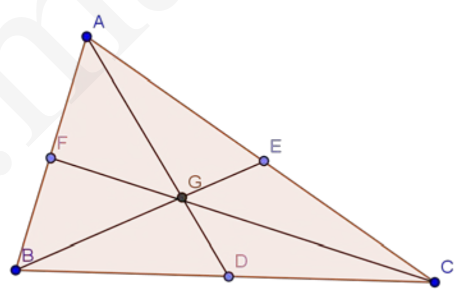






b) **Mediana** ***= segmentul determinat de un vârf al triunghiului și mijlocul laturii opuse.***

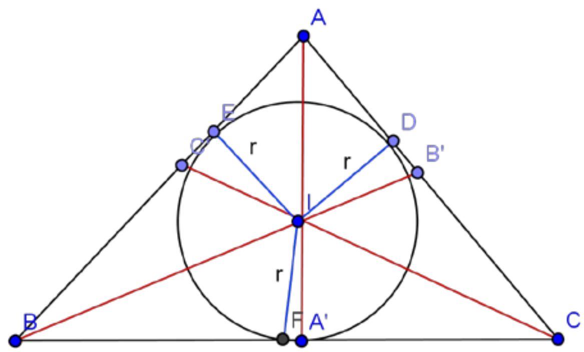
- Intersecția medianelor este centrul de greutate al triunghiului



$$\left.\begin{array}{l} G \in AD \\ [AD] - \textit{mediană} \end{array}\right\} \Rightarrow GD = \frac{1}{3} AD \; si \;\; AG = \frac{2}{3} AD$$

c) **Bisectoarea** ***(unui unghi propriu) = semidreapta cu originea în vârful unghiului, situata în interiorul lui, astfel încât cele două unghiuri formate de ea cu laturile unghiului inițial să fie congruente.***

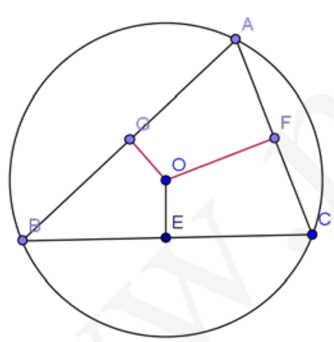
- intersectia bisectoarelor este centrul cercului înscris în triunghi



$r = \dfrac{S}{p}$ S - aria triunghiului
p - semiperimetrul
r - raza cercului înscris în triunghi

d) **Mediatoarea (unui segment) = *dreapta perpendiculară dusă prin mijlocul segmentului dat***

- Intersecția mediatoarelor laturilor unui triunghi este centrul cercului circumscris triunghiului



$$OA = OB = OC = R$$

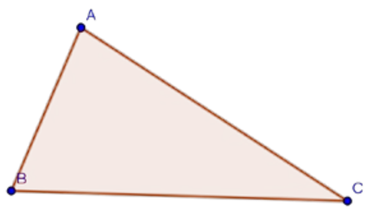
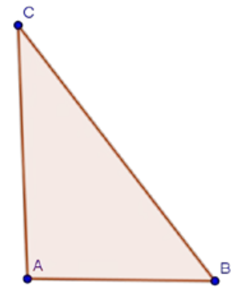
$$R = \frac{a \cdot b \cdot c}{4S}$$

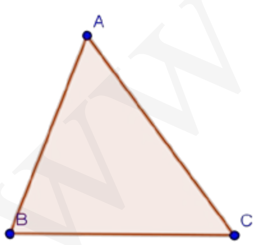
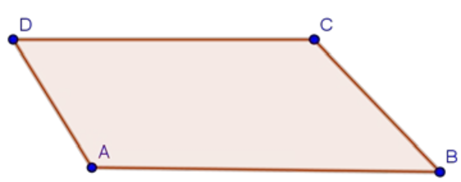
R – raza cercului circumscris triunghiului
a, b, c – laturile triunghiului
S – aria triunghiului

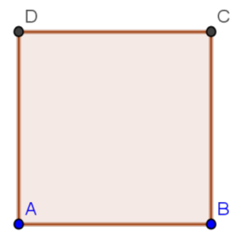
3. Criterii de congruență pentru triunghiul oarecare
L.U.L, U.L.U, L.L.L. , L.U.U.*

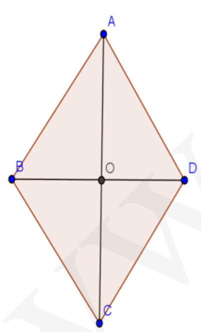
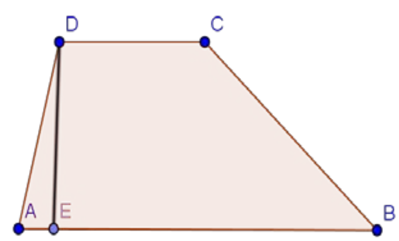
4. Cazurile de congruență pentru triunghiurile dreptunghice
C.C., C.U., I.U., I.C.

ARII

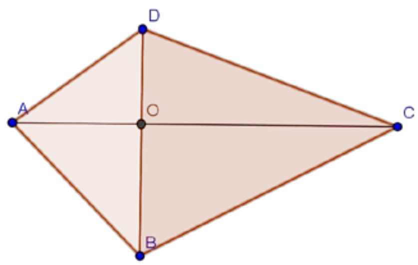
1. Arie triunghiul oarecare	2. Arie triunghiul dreptunghic
	
$A_{\Delta oarecare} = \dfrac{b \cdot h}{2} = \dfrac{l_1 \cdot l_2 \cdot \sin u}{2}$ **Formula lui Heron** $A_{\Delta oarecare} = \sqrt{p(p-a)(p-b)(p-c)},$ $unde\ p = \dfrac{a+b+c}{2}$	$A_{\Delta dreptunghic} = \dfrac{c_1 \cdot c_2}{2}$ $h_{\Delta dreptunghic} = \dfrac{c_1 \cdot c_2}{ip}$

3. Arie triunghiul echilateral	4. Arie paralelogram
	
$A_{\Delta echilateral} = \dfrac{l^2\sqrt{3}}{4}$ $h_{\Delta echilateral} = \dfrac{l\sqrt{3}}{2}$	$A_{paralelogram} = b \cdot h$ $A_{paralelogram} = l_1 \cdot l_2 \cdot \sin u$ $P_{paralelogram} = 2(AB+BC)$

5. Arie dreptunghi	6. Arie patrat
	
$A_{dreptunghi} = L \cdot l$ $P_{dreptunghi} = 2(L+l)$	$A_{patrat} = l^2$ $P_{patrat} = 4l$ $d_{patrat} = l\sqrt{2}$

7. Arie romb	8. Arie trapez
	
$A_{romb} = \frac{d_1 \cdot d_2}{2} = b \cdot h$ $A_{romb} = l^2 \cdot \sin A$ $P_{romb} = 4l$	$A_{trapez} = \frac{(B+b)\cdot h}{2} = l_m \cdot h$ $l_m = \frac{B+b}{2} (linia\ mijlocie)$ $P_{trapez} = AB + BC + CD + AD$

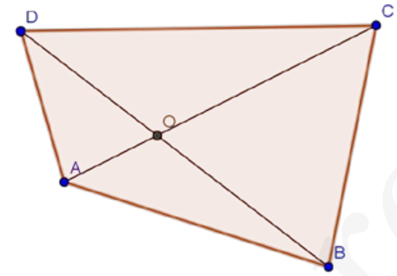
9. Arie patrulater ortodiagonal



$$A_{patrulater\,ortodiagonal} = \frac{d_1 \cdot d_2}{2}$$

$$P_{patrulater\,ortodiagonal} = AB + BC + CD + AD$$

10. Arie patrulater convex

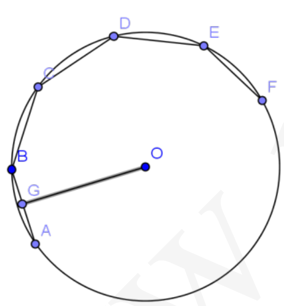


$$A_{patrulater\,convex} = \frac{d_1 \cdot d_2 \cdot \sin\alpha}{2}, \alpha = m(\widehat{d_{1,} d_2})$$

$$A_{patrulater\,convex} = A_{ABD} + A_{BDC}$$

$$P_{patrulater\,ortodiagonal} = AB + BC + CD + AD$$

11. Arie poligon regulat



$$A_{poligon\,regulat} = \frac{P_n \cdot a_n}{2}$$

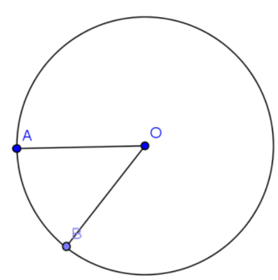
$$l_n = 2R\sin\frac{n^o}{2}$$

$$a_n = R\cos\frac{n^o}{2}$$

$$m(\overset{\frown}{AB}) = n^o$$

$$m(\sphericalangle ABC) = \frac{180(n-2)}{n}$$

12. Arie disc



$$A_{disc} = \pi R^2$$

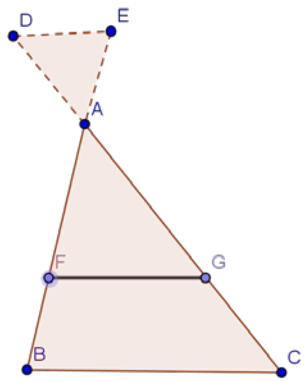
$$L_{cerc} = 2\pi R$$

$$l_{arc} = \frac{\pi R n}{180}$$

$$A_{sector} = \frac{\pi R^2 n}{360}$$

RELAȚII METRICE ÎN TRIUNGHI

1. Teorema lui Thales



$$\left.\begin{array}{l}\triangle ABC \\ FG \parallel BC\end{array}\right\} \Rightarrow \frac{FA}{FB} = \frac{GA}{GC}$$

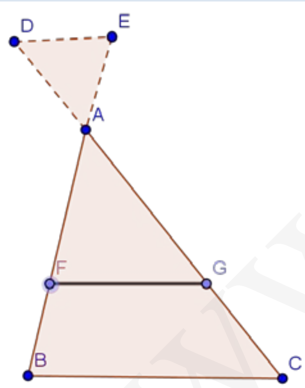
$$\left.\begin{array}{l}\triangle ABC \\ DE \parallel BC\end{array}\right\} \Rightarrow \frac{EA}{EB} = \frac{DA}{DC}$$

Reciproca Teoremei lui Thales

$$Daca\,\frac{FA}{FB} = \frac{GA}{GC} \Rightarrow FG \parallel BC$$

$$Daca\,\frac{EA}{EB} = \frac{DA}{DC} \Rightarrow DE \parallel BC$$

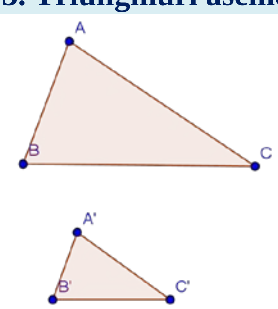
2. Teorema fundamental a asemanarii



$$\left.\begin{array}{l}\triangle ABC \\ FG \parallel BC\end{array}\right\} \Rightarrow \triangle AFG \sim \triangle ABC$$

$$\left.\begin{array}{l}\triangle ABC \\ DE \parallel BC\end{array}\right\} \Rightarrow \triangle AED \sim \triangle ABC$$

3. Triunghiuri asemenea



$\triangle ABC \sim \triangle A'B'C' \Leftrightarrow$

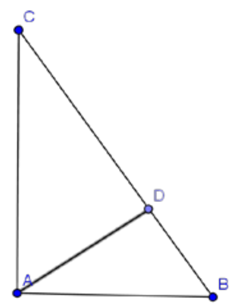
$\frac{AB}{A'B'} = \frac{AC}{A'C'} = \frac{BC}{B'C'}$,

$\sphericalangle A \equiv \sphericalangle A', \sphericalangle B \equiv \sphericalangle B', \sphericalangle C \equiv \sphericalangle C'$

4. Cazuri de asemanare

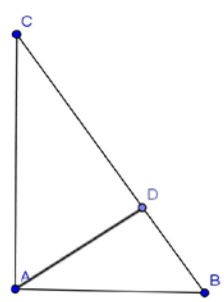
1) U.U.
2) $\frac{AB}{A'B'} = \frac{AC}{A'C'}$ si $\sphericalangle A \equiv \sphericalangle A'$
3) $\frac{AB}{A'B'} = \frac{AC}{A'C'} = \frac{BC}{B'C'}$

5. Teorema catetei



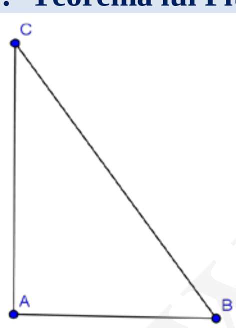
$$\left.\begin{array}{l}\triangle ABC\\ m(\sphericalangle A)=90^o\\ AD\perp BC\end{array}\right\}\overset{T.Catetei}{\Rightarrow}\left\{\begin{array}{l}AB^2=BD\cdot BC\\ AC^2=CD\cdot CB\end{array}\right.$$

6. Teorema înălțimii



$$\left.\begin{array}{l}\triangle ABC\\ m(\sphericalangle A)=90^o\\ AD\perp BC\end{array}\right\}\overset{T.Inaltimii}{\Rightarrow} AD^2=BD\cdot DC$$

7. Teorema lui Pitagora



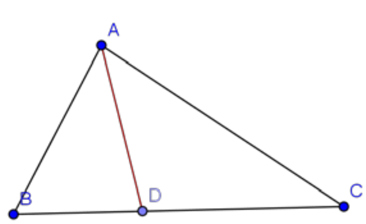
$$\left.\begin{array}{l}\triangle ABC\\ m(\sphericalangle A)=90^o\end{array}\right\}\overset{T.Pitagora}{\Rightarrow} BC^2=AB^2+AC^2$$

Reciproca Teorema lui Pitagora

Dacă în $\triangle ABC$ avem BC > AC > AB si

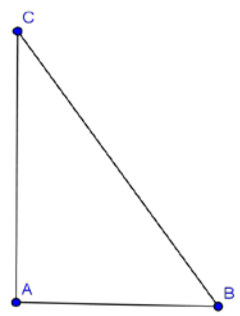
$BC^2=AB^2+AC^2 \Rightarrow \triangle ABC\ dreptunghic, m(\sphericalangle A)=90^o$

8. Teorema bisectoarei



$$\left.\begin{array}{l}\triangle ABC\\ (AD)\,bisectoare\end{array}\right\}\Rightarrow \frac{AB}{AC}=\frac{BD}{DC}$$

ELEMENTE DE TRIGONOMETRIE



$$\sin x = \frac{cateta\ opusă}{ipotenuză}$$

$$\cos x = \frac{cateta\ alăturată}{ipotenuză}$$

$$tgx = \frac{cateta\ opusă}{cateta\ alăturată}$$

$$ctgx = \frac{cateta\ alăturată}{cateta\ opusă}$$

	30^o	45^o	60^o
sinx	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cosx	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tgx	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$
ctgx	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$

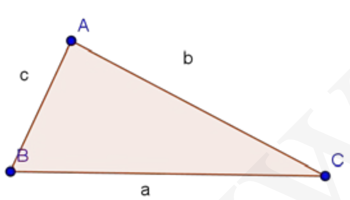
TEOREMA UNGHIULUI DE 30^O

Într-un triunghi dreptunghic cateta opusă unghiului de 30^0 este jumatate din ipotenuză

TEOREMA – Mediana in triunghiul dreptunghic

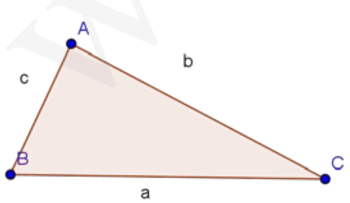
Într-un triunghi dreptunghic mediana dusă din vârful unghiului drept este jumatate din ipotenuză

TEOREMA COSINUSURILOR *(se aplica în triunghiul oarecare)*



$$a^2 = b^2 + c^2 - 2bc \cdot \cos \hat{A}$$
$$b^2 = a^2 + c^2 - 2ac \cdot \cos \hat{B}$$
$$c^2 = a^2 + b^2 - 2ab \cdot \cos \hat{C}$$

TEOREMA SINUSURILOR *(se aplică în triunghiul oarecare)*



$$\frac{a}{\sin \hat{A}} = \frac{b}{\sin \hat{B}} = \frac{c}{\sin \hat{C}} = 2R$$

Material realizat de Andrei Octavian Dobre– www.mateinfo.ro

(Profesor de matematică – Ploiești)

Contact: office@mateinfo.ro ; dobre.andrei@yahoo.com