

Numere complexe

Un număr complex este de forma $z=a+ib, a, b \in \mathfrak{R}$, unde $i^2 = -1$: $\text{Re}(z) = a, \text{Im}(z) = b$

Operatii cu numere complexe.

Fie $z_1 = a_1 + ib_1$ si $z_2 = a_2 + ib_2$

$$z_1 + z_2 = (a_1 + a_2) + i(b_1 + b_2):$$

$$z_1 z_2 = (a_1 a_2 - b_1 b_2) + i(a_1 b_2 + a_2 b_1);$$

$$\frac{1}{z} = \frac{a}{a^2 + b^2} - i \frac{b}{a^2 + b^2};$$

$$\frac{z_2}{z_1} = \frac{a_1 a_2 + b_1 b_2}{a_1^2 + b_1^2} - i \frac{a_1 b_2 - a_2 b_1}{a_1^2 + b_1^2}.$$

Numere complexe conjugate

$\bar{z} = \overline{a+ib} = a - ib$ se numeste conjugatul numarului complex $z=a+ib$

Avem a) $a = \frac{z + \bar{z}}{2}; b = \frac{z - \bar{z}}{2i}$ b) $z \in \mathfrak{R}$ daca si numai daca $z = \bar{z}$

c) z este imaginar daca si numai daca $\bar{z} = -z$ d) $\overline{\alpha z} = \alpha, \forall \alpha \in \mathfrak{R}$

Fie $z=a+ib$ si $z'=a'+ib'$ doua numere complexe. Avem:

a) $\overline{z + z'} = \bar{z} + \bar{z}';$ b) $\overline{z \cdot z'} = \bar{z} \cdot \bar{z}';$ c) $\frac{z}{z'} = \frac{z \cdot \bar{z}'}{a'^2 + b'^2}, z' \neq 0$