BAREM DE EVALUARE ŞI DE NOTARE

Varianta 12

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- Pentru orice soluţie corectă, chiar dacă este diferită de cea din barem, se acordă punctajul corespunzător.
- Nu se acordă fracţiuni de punct, dar se pot acorda punctaje intermediare pentru rezolvări parţiale, în limitele punctajului indicat în barem.
- Se acordă 10 puncte din oficiu. Nota finală se calculează prin împărţirea punctajului obţinut la 10.

SUBIECTUL I (30 de puncte)

1.	$x-3, x, x+1 \Rightarrow x^2=(x-3)(x+1)$ $2x+3=0 \Rightarrow x=\frac{-3}{2}$	3p 2p
2.	$x+1+\sqrt{x-3}=x+1, x\in[3,\infty)$ $\sqrt{x-3}=0 \Rightarrow x=3\in[3,\infty)$	3p 2p
3.	$(f\circ f)(x)=f(f(x))=3(3x-2)-2=9x-8$ $(f\circ f)(x)-f(x)=0 \Leftrightarrow 9x-8-(3x-2)=0$ $x=1$	2p 2p 1p
4.	nr dreptelor=C_{10}^2 $\frac{10!}{2!\,(10-2)!}=45$	2p 3p
5.	$m_{AB}=\frac{y_B-y_A}{x_B-x_A} \Rightarrow m_{AB}=2 \Rightarrow m_{mediat.}=-\frac{1}{2}$ $Mmij.[AB] \Rightarrow M\left(\frac{x_A+x_B}{2},\frac{y_A+y_B}{2}\right) \Rightarrow M\left(\frac{5}{2},4\right)$ $ec.mediat: y-y_M=m_{mediat.}(x-x_M) \Rightarrow 2x+4y-21=0$	1p 2p 2p
6.	$4\in\left(\pi,\frac{3\pi}{2}\right) \Rightarrow \cos 4<0$ $5\in\left(\frac{3\pi}{2},2\pi\right) \Rightarrow \cos 5>0$ $\cos 4<\cos 5$	2p 1p 2p

SUBIECTUL al II-lea (30 de puncte)

1. a)	$A(n)=\begin{pmatrix} e^{\ln n} & \ln e \\ \ln 1 & \ln e \end{pmatrix}=\begin{pmatrix} n & 1 \\ 0 & 1 \end{pmatrix}, A(3)=\begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}, A(4)=\begin{pmatrix} 4 & 1 \\ 0 & 1 \end{pmatrix}$	3p
	$A(3)A(4)=\begin{pmatrix} 12 & 4 \\ 0 & 1 \end{pmatrix}\Rightarrow tr\left(A(3)A(4)\right)=13$	2p
b)	$A(3)=\begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}\Rightarrow A^3(3)=\begin{pmatrix} 27 & 13 \\ 0 & 1 \end{pmatrix}$	3p
	$\det A^3(3)=27$	2p
c)	$A(1)=\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}; A^2(1)=\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}; A^3(1)=\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}; A^n(1)=\begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}, n\in\mathbb{N}^*$	
	Etapa de verificare $n=1, A^1(1)=\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$	
	Etapa de demonstratie $A^{n+1}(1)=\begin{pmatrix} 1 & n+1 \\ 0 & 1 \end{pmatrix}; A^{n+1}(1)=A^n(1)A(1)=\begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}=\begin{pmatrix} 1 & n+1 \\ 0 & 1 \end{pmatrix}$	
	$A^{2014}(1)=\begin{pmatrix} 1 & 2014 \\ 0 & 1 \end{pmatrix}$	
2. a)	$g=0\Rightarrow x^2+\hat{3}x+\hat{2}=\hat{0}$	1p
	$x=\hat{3}$	2p
	$x=\hat{4}$	2p
b)	$g \mid f\Rightarrow f=g\cdot c\Rightarrow f(\hat{3})=\hat{0}, f(\hat{4})=\hat{0}$	2p
	$\hat{3}^4=\hat{1}, \hat{4}^4=\hat{1}$	2p
	$\hat{1}+a=\hat{0}\Rightarrow a=\hat{4}$	1p
c)	$f=x^4+\hat{1}$	1p
	$\hat{a}^4\in\{\hat{0},\hat{1}\}, \forall a\in\mathbb{Z}_5[x]$	2p
	$f(\hat{a})\in\{\hat{1},\hat{2}\}, \forall a\in\mathbb{Z}_5[x]$	2p

SUBIECTUL al III-lea (30 de puncte)

1. a)	$f'(x)=\left(\frac{x+3}{x-3}\right)'=\frac{x-3-x-3}{(x-3)^2}=\frac{-6}{(x-3)^2}$	3p
	$\lim\limits_{x\to 0}\frac{f(x)-f(0)}{x}=f'(0)=\frac{-6}{9}$	2p
b)	$\lim\limits_{x\to\infty}(f(x))^x=\lim\limits_{x\to\infty}\left(\frac{x+3}{x-3}\right)^x\overset{1^\infty}{=}\lim\limits_{x\to\infty}\left(1+\frac{6}{x-3}\right)^x=$	3p
	$\lim\limits_{x\to\infty}\left(1+\frac{6}{x-3}\right)^{\frac{x-3}{6}\frac{6}{x-3}x}=e^6$	2p
c)	$f'(x)=\frac{-6}{(x-3)^2}<0,\forall x\in\mathbb{R}\setminus\{3\}\Rightarrow f$ strict descrescatoare pe $\mathbb{R}\setminus\{3\}$	1p
	$\lim\limits_{x\to-\infty}f(x)=\lim\limits_{x\to-\infty}f(x)=1,\lim\limits_{\substack{x\to3\\x<3}}f(x)=-\infty,\lim\limits_{\substack{x\to3\\x>3}}f(x)=\infty$	2p
	Daca $m=1$ ecuatia nu are solutii;daca $m\in(-\infty,1)$ ecuatia are o solutie $x_0\in(-\infty,3)$; daca $m\in(1,\infty)$ ecuatia are o solutie $x_0\in(3,\infty)$	2p
2. a)	$I_n=\int_e^{e^2}e^{\ln x^n}\ln x dx=\int_e^{e^2}x^n\ln x dx$	2p
	$I_1=\int_e^{e^2}x\ln x dx=\frac{x^2}{2}\ln x\Big\vert_e^{e^2}-\int_e^{e^2}\frac{x^2}{2}\frac{1}{x}dx=$	2p
	$e^4-\frac{e^2}{2}-\frac{x^2}{4}\Big\vert_e^{e^2}=\frac{3e^4-e^2}{4}$	1p
b)	$I_n=\int_e^{e^2}x^n\ln x dx=\frac{x^{n+1}}{n+1}\ln x\Big\vert_e^{e^2}-\int_e^{e^2}\frac{x^{n+1}}{n+1}\frac{1}{x}dx=$	1p
	$\frac{2e^{2n+2}-e^{n+1}}{n+1}-\frac{x^{n+1}}{(n+1)^2}\Big\vert_e^{e^2}=$	1p
	$\frac{(n+1)\left(2e^{2n+2}-e^{n+1}\right)-\left(e^{2n+2}-e^{n+1}\right)}{(n+1)^2}=\frac{(2n+1)e^{2n+2}-ne^{n+1}}{(n+1)^2}$	1p 2p
c)	$I_0=e^2,I_1=\frac{3e^4-e^2}{4},I_2=\frac{5e^6-2e^3}{9}$	3p
	$5e^2e^2+20e^2\frac{3e^4-e^2}{4}-6e^3=27\frac{5e^6-2e^3}{9}$	1p
	$5e^4+15e^6-5e^4-6e^3=15e^6-6e^3$	1p
