Limită

$$\lim_{x\to 0}\left(\frac{1}{x^2}-\operatorname{ctg}^2 x\right)=\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{\cos^2 x}{\sin^2 x}\right)=\lim_{x\to 0}\left(\frac{\sin^2 x-x^2\cos^2 x}{x^2\sin^2 x}\right)=$$

$$=\lim_{x\to 0}\left(\underbrace{\frac{\sin x+x\cos x}{x}}_{E_1(x)}\cdot\underbrace{\frac{\sin x-x\cos x}{x\sin^2 x}}_{E_2(x)}\right)=\lim_{x\to 0}\left(E_1(x)\cdot E_2(x)\right).$$

Avem

$$\lim_{x\to 0}E_1(x)=\lim_{x\to 0}\frac{\sin x+x\cos x}{x}=\lim_{x\to 0}\left(\frac{\sin x}{x}+\frac{x}{x}\cos x\right)=\lim_{x\to 0}\left(\frac{\sin x}{x}+\cos x\right)=1+1=2.$$

Pentru $\lim\limits_{x\to 0}E_2(x)$ aplicăm *l'Hôpital.*

$$\lim_{x\to 0}E_2(x)=\lim_{x\to 0}\frac{\sin x-x\cos x}{x^2\sin x}\overset{\frac{0}{0}}{\underset{l'H}{=}}\lim_{x\to 0}\frac{\cos x-\cos x+x\sin x}{2x\sin x+x^2\cos x}=\lim_{x\to 0}\frac{x\sin x}{2x\sin x+x^2\cos x}=$$

$$=\lim_{x\to 0}\frac{\cancel{x}\sin x}{\cancel{x}\left(2\sin x+x\cos x\right)}=\lim_{x\to 0}\frac{\sin x}{2\sin x+x\cos x}\overset{\frac{0}{0}}{\underset{l'H}{=}}\lim_{x\to 0}\frac{\cos x}{3\cos x-x\sin x}=\frac{1}{3}.$$

Prin urmare,

$$\lim_{x\to 0}\left(E_1(x)\cdot E_2(x)\right)=2\cdot\frac{1}{3}=\frac{2}{3}.$$