

CALCULUL LIMITELOR DE FUNCȚII

Elev:.....

PROF. STAN ADRIAN

CAZUL DE NEDETERMINARE $\frac{\infty}{\infty}$

Grup Școlar „ Costin

Nenitescu”,Buzău

Exerciții rezolvate:

1. $\lim_{x \rightarrow \infty} \frac{-2x+3}{\sqrt{x^2+5}} = \lim_{x \rightarrow \infty} \frac{x(-2+\frac{3}{x})}{x\sqrt{1+\frac{5}{x^2}}} = -2;$
2. $\lim_{x \rightarrow \infty} \frac{3x+5}{\sqrt{4x^2+6}} = \lim_{x \rightarrow \infty} \frac{3x+5}{\sqrt{4x^2(1+\frac{6}{4x^2})}} = \lim_{x \rightarrow \infty} \frac{x(3+\frac{5}{x^2})}{2|x|\sqrt{(1+\frac{6}{4x^2})}} = \frac{-3}{2};$
3. $\lim_{x \rightarrow \infty} \frac{3x+8}{2x^2+5x} = \lim_{x \rightarrow \infty} \frac{x(3+\frac{8}{x})}{x^2(2+\frac{5}{x})} = 0;$
4. $\lim_{x \rightarrow \infty} \frac{(x+1)^3 - x^3}{3x^2+2x} = \lim_{x \rightarrow \infty} \frac{x^3+3x^2+3x+1-x^3}{3x^2+2x} = \lim_{x \rightarrow \infty} \frac{3x^2+3x+1}{3x^2+2x} = 1$
5. $\lim_{x \rightarrow \infty} \frac{\ln(x^2+x)}{\ln(x^4+2x)} = \lim_{x \rightarrow \infty} \frac{\ln x^2(1+\frac{1}{x})}{\ln x^4(1+\frac{2}{x^3})} = \lim_{x \rightarrow \infty} \frac{2\ln x + \ln(1+\frac{1}{x})}{4\ln x + \ln(1+\frac{2}{x^3})} = \frac{1}{2};$
6. $\lim_{x \rightarrow \infty} \frac{2^x+3^x}{5 \cdot 3^x - 2^x} = \lim_{x \rightarrow \infty} \frac{3^x \left[\left(\frac{2}{3}\right)^x + 1 \right]}{3^x \left[5 - \left(\frac{2}{3}\right)^x \right]} = \frac{1}{5};$
7. $\lim_{x \rightarrow \infty} \frac{2x+5}{\sqrt{3x^2+9x+6}} = \lim_{x \rightarrow \infty} \frac{x\left(2+\frac{5}{x}\right)}{\sqrt{x^2\left(3+9 \cdot \frac{1}{x} + \frac{6}{x^2}\right)}} = \lim_{x \rightarrow \infty} \frac{x\left(2+\frac{5}{x}\right)}{-x \cdot \sqrt{3+\frac{9}{x} + \frac{6}{x^2}}} = \frac{2}{-3}$

Probleme propuse:

8. $\lim_{x \rightarrow \infty} \frac{x^2+x+1}{\sqrt{x^4+1}};$
9. $\lim_{x \rightarrow \infty} \frac{3x^3-5x^2+2}{2-3x};$
10. $\lim_{x \rightarrow \infty} \frac{-3x^5+6x^2+2}{1-3x^2};$
11. $\lim_{x \rightarrow \infty} \frac{-3x+4}{\sqrt{3x^2-8}};$
12. $\lim_{x \rightarrow \infty} \frac{\ln(x^2-4x)}{\ln(x^2-5x)};$
13. $\lim_{x \rightarrow \infty} \frac{\ln(3^{2x}+5^{2x})}{\ln(3^x+5^x)};$
14. $\lim_{x \rightarrow \infty} \frac{2x+5}{|x-4|};$
15. $\lim_{x \rightarrow \infty} \frac{4x+6}{\sqrt{9x^2-7}};$
16. $\lim_{x \rightarrow \infty} \frac{\ln(1+4^{2x})}{x+3};$
17. $\lim_{x \rightarrow \infty} \frac{3^{3x}+4^{3x}}{2^{5x}+4^{5x}};$

Rezolvă singur exercițiile:

Temă:

- 1) $\lim_{x \rightarrow \infty} \frac{4x+6}{5x^2+5x};$ 2) $\lim_{x \rightarrow \infty} \frac{-x^3+x^2}{2x^2+4};$ 3) $\lim_{x \rightarrow \infty} \frac{-7x^2+3x-5}{4x^2-1};$ 4) $\lim_{x \rightarrow \infty} \frac{2x^4-x^2}{3x^3+x+2};$
- 5) $\lim_{x \rightarrow \infty} \frac{6x^3-5x+11}{-2x^3+4x^2-\sqrt{6}};$ 6) $\lim_{x \rightarrow \infty} \frac{-x+3}{\sqrt{2x^2-5}};$ 7) $\lim_{x \rightarrow \infty} \frac{x+2}{|x-1|};$ 8) $\lim_{x \rightarrow \infty} \frac{x+2}{|x-1|};$
- 9) $\lim_{x \rightarrow \infty} \frac{|x|}{\sqrt{x^2+4}};$ 10) $\lim_{x \rightarrow \infty} \frac{6x}{\sqrt{9x^2+5}};$ 11) $\lim_{x \rightarrow \infty} \frac{4x^3-4x+4}{\sqrt{x^6-1}};$ 12) $\lim_{x \rightarrow \infty} \frac{x^2+1}{\sqrt{x^2-1}};$
- 13) $\lim_{x \rightarrow \infty} \frac{3x^2}{\sqrt{4x^2+4}};$ 14) $\lim_{x \rightarrow \infty} \frac{-2x+8}{\sqrt{x^2-7}};$ 15) $\lim_{x \rightarrow \infty} \frac{\ln(x^3+3x)}{\ln(x^5-5x)};$ 16) $\lim_{x \rightarrow \infty} \frac{\ln(e^{3x}+1)}{\ln(e^x+1)};$

Răspunsuri: 1. 0; 2. ∞ ; 3. $-\frac{7}{4}$; 4. ∞ ; 5. -3; 6. 0; 7. -1; 8. 1; 9. 1; 10. -2; 11. -4; 12. ∞ ; 13. ∞ ; 14. 2; 15. $\frac{3}{5}$; 16. 3.

CALCULUL LIMITELOR DE FUNCȚII

Elev:.....

PROF. STAN ADRIAN

CAZUL DE NEDETERMINARE $\infty - \infty$
Nenitescu”, Buzău

Grup Școlar „ Costin

Exerciții rezolvate:

$$1. \lim_{x \rightarrow \infty} (\sqrt{4x^2 + 1} - 2x) = \lim_{x \rightarrow \infty} \frac{4x^2 + 1 - 4x^2}{\sqrt{4x^2 + 1} + 2x} =$$

$$\lim_{x \rightarrow \infty} \frac{1}{2x(\sqrt{1 + \frac{1}{4x^2}} + 1)} = 0;$$

$$2. \lim_{x \rightarrow -\infty} x(\sqrt{9x^2 + 5} + 3x) = \lim_{x \rightarrow -\infty} -x(\sqrt{9x^2 + 5} - 3x) =$$

$$= \lim_{x \rightarrow \infty} \left(-x \cdot \frac{9x^2 + 5 - 9x^2}{\sqrt{9x^2 + 5} + 3x} \right) = \lim_{x \rightarrow \infty} \frac{-5x}{3x(\sqrt{1 + \frac{5}{9x^2}} + 1)} = \frac{-5}{3};$$

$$3. \lim_{x \rightarrow \infty} (\sqrt[3]{x^3 + 4x} - \sqrt[3]{x^3 + 2}) =$$

$$= \lim_{x \rightarrow \infty} \frac{x^3 + 4x - x^3 + 2}{\sqrt[3]{(x^3 + 4x)^2} + \sqrt[3]{(x^3 + 4x) \cdot (x^3 + 2)} + \sqrt[3]{(x^3 + 2)^2}} =$$

$$= \lim_{x \rightarrow \infty} \frac{4x + 2}{x^3 \left(\sqrt[3]{1 + \frac{8}{x^2} + \frac{16}{x^4}} + \sqrt[3]{\left(1 + \frac{4}{x^2}\right) \cdot \left(1 + \frac{2}{x^2}\right)} + \sqrt[3]{1 + \frac{4}{x^2} + \frac{4}{x^4}} \right)} = 0;$$

$$4. \lim_{\substack{x \rightarrow 3 \\ x)3}} \left(\frac{5x}{x-3} - \frac{2}{x(x-3)} \right) = \lim_{\substack{x \rightarrow 3 \\ x)3}} \left(\frac{5x^2 - 2}{x(x-3)} \right) = \infty;$$

Probleme propuse:

$$5. \lim_{x \rightarrow \infty} x (\sqrt{x^2 + 4} - x)$$

$$6. \lim_{x \rightarrow -\infty} (\sqrt{x^2 + 2x} + x)$$

$$7. \lim_{x \rightarrow \infty} (\sqrt{x^2 + 3x} - \sqrt{x^2 - 3})$$

$$8. \lim_{x \rightarrow \infty} (x + 2 - \sqrt{x^2 + 4x + 1})$$

$$9. \lim_{x \rightarrow -\infty} \left(\frac{2x^3}{x^2 + 4} - 2x \right)$$

$$10. \lim_{\substack{x \rightarrow 2 \\ x)2}} \left(\frac{3x}{x-2} - \frac{1}{x^2 - 4} \right)$$

$$11. \lim_{x \rightarrow \infty} (\ln(3x + 2) - \ln(2x - 1))$$

$$12. \lim_{x \rightarrow \infty} \left(\frac{\sqrt{2x^2 - 3x} - \sqrt{2x^2 - 5}}{\sqrt{x^2 + 4x - 2} - \sqrt{x^2 + 3x - 3}} \right)$$

Rezolvă singur exercițiile:

Temă:

$$1. \lim_{x \rightarrow \infty} (\sqrt{x^2 + 1} - 2x); \quad 2. \lim_{x \rightarrow \infty} 2x (\sqrt{x^2 + 3} - x); \quad 3. \lim_{x \rightarrow -\infty} (\sqrt{x^2 + 3x} + x);$$

$$4. \lim_{x \rightarrow \infty} (\sqrt{x^2 + 2x - 1} - \sqrt{x^2 + x - 1}); \quad 5. \lim_{x \rightarrow \infty} (\sqrt[3]{x^3 + x^2 + 1} - \sqrt[3]{x^3 - x^2 + 1});$$

$$6. \lim_{x \rightarrow \infty} (\sqrt{x+1} - 2\sqrt{x+2} + \sqrt{x+3}); \quad 7. \lim_{x \rightarrow -\infty} (\sqrt{x^2 - x + 1} + x);$$

$$8. \lim_{x \rightarrow \infty} \frac{(x - \sqrt{2x^2 - 3x + 5})}{x}; \quad 9. \lim_{x \rightarrow \infty} (x - 3 - \sqrt{x^2 - 6x + 5}); \quad 10. \lim_{x \rightarrow -\infty} \left(\frac{5x^3}{x^2 + 1} - 5x \right);$$

$$11. \lim_{\substack{x \rightarrow 1 \\ x)1}} \left(\frac{2x}{x-1} - \frac{5}{x^2 - 1} \right); \quad 12. \lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{1}{\cos x} - \frac{1}{\sin 2x} \right); \quad 13. \lim_{x \rightarrow 2} \left(\frac{1}{x(x-2)^2} - \frac{1}{x^2 - 3x + 2} \right);$$

$$14. \lim_{x \rightarrow \infty} (\ln(4x - 5) - \ln(4x + 3)); \quad 15. \lim_{x \rightarrow \infty} (\ln(2x^3 - 6) - \ln(3x^2 + 2));$$

Răspunsuri: 1. $-\infty$; 2. 6; 3. $-\frac{3}{2}$; 4. $\frac{1}{2}$; 5. $\frac{2}{3}$; 6. 0; 7. $\frac{1}{2}$; 8. $1-\sqrt{2}$; 9. 0; 10. 0; 11. $-\infty$; 12. $-\infty$; 13. ∞ ; 14. 0; 15. ∞ .

CALCULUL LIMITELOR DE FUNCȚII

Elev:.....

PROF. STAN ADRIAN

CAZUL DE NEDETERMINARE 1^∞
Nenitescu”,Buzău

Grup Școlar „ Costin

Exerciții rezolvate:

1. $\lim_{x \rightarrow 0} (1 + 5x)^{\frac{3}{x}} = \lim_{x \rightarrow 0} \left[(1 + 5x)^{\frac{1}{5x}} \right]^{\frac{15x}{x}} = e^{15}$;
 2. $\lim_{x \rightarrow 3} (7 - 2x)^{\frac{1}{x-3}} = \lim_{x \rightarrow 3} \left[(1 + 6 - 2x)^{\frac{1}{6-2x}} \right]^{\frac{6-2x}{x-3}} = e^{\lim_{x \rightarrow 3} \frac{-2(x-3)}{x-3}} = e^{-2} = \frac{1}{e^2}$;
 3. $\lim_{x \rightarrow \infty} \left(1 - \frac{1}{4x} \right)^{8x} = \lim_{x \rightarrow \infty} \left[\left(1 + \frac{-1}{4x} \right)^{-4x} \right]^{\frac{8x}{-4x}} = e^{-2} = \frac{1}{e^2}$;
 4. $\lim_{x \rightarrow \infty} \left(\frac{2x^2 - x + 11}{2x^2 + 3x - 1} \right)^{\frac{x^2}{2x-1}} = \lim_{x \rightarrow \infty} \left[\left(1 + \frac{-4x+12}{2x^2+3x-1} \right)^{\frac{2x^2+3x-1}{-4x+12}} \right]^{\frac{x^2(-4x+12)}{(2x^2+3x-1)(2x-1)}} = e^{-1} = \frac{1}{e}$
 5. $\lim_{x \rightarrow \infty} \left(\frac{x^2 + x + 1}{x^2 - x + 1} \right)^{\sqrt{x^2+4}} = \lim_{x \rightarrow \infty} \left[\left(1 + \frac{2x}{x^2 - x + 1} \right)^{\frac{x^2-x+1}{2x}} \right]^{\frac{2x\sqrt{x^2+4}}{x^2-x+1}} =$
- $$e^{\lim_{x \rightarrow \infty} \frac{2x^2 \sqrt{1 + \frac{4}{x^2}}}{x^2 - x + 1}} = e^2$$

Exerciții propuse:

6. $\lim_{x \rightarrow 0} (1 + 3x)^{\frac{5}{2x}}$;
7. $\lim_{x \rightarrow 2} (15 - 7x)^{\frac{1}{3x-6}}$;
8. $\lim_{x \rightarrow \infty} \left(1 - \frac{1}{3x} \right)^{\frac{2x^2+3}{x+1}}$;
9. $\lim_{x \rightarrow 0} (1 + \sin 2x)^{\frac{1}{6x}}$;
10. $\lim_{x \rightarrow 0} (1 + 2x^2 + 4x)^{\frac{1}{tg 2x}}$;
11. $\lim_{x \rightarrow \infty} \left(\frac{2x + 4}{2x + 3} \right)^{\frac{x^2+3}{2x}}$;
12. $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{\frac{\sin x}{x - \sin x}}$;
13. $\lim_{x \rightarrow 0} \left(\frac{4^x + 8^x}{2} \right)^{\frac{1}{x}}$;
14. $\lim_{x \rightarrow 0} (1 + tg^2 \sqrt{x})^{\frac{1}{3x}}$;
15. $\lim_{x \rightarrow 0} \left(\frac{1 + \sin x}{1 - \sin x} \right)^{\frac{1}{x}}$;

Rezolvă singur exercițiile:

Temă:

1. $\lim_{x \rightarrow 0} (1 + 6x)^{\frac{2}{3x}}$; 2. $\lim_{x \rightarrow 3} (13 - 4x)^{\frac{1}{x-3}}$; 3. $\lim_{x \rightarrow \infty} \left(1 - \frac{1}{6x-3} \right)^{\frac{9x+2}{3}}$; 4. $\lim_{x \rightarrow 0} (1 + \sin 3x)^{\frac{1}{3x}}$;
5. $\lim_{x \rightarrow \frac{\pi}{2}} (\sin x)^{\frac{1}{2x-\pi}}$; 6. $\lim_{x \rightarrow \infty} \left(\frac{3x+1}{3x-1} \right)^{2x-3}$; 7. $\lim_{x \rightarrow -\infty} \left(\frac{x^2+x+1}{x^2-x+1} \right)^{x^2}$; 8. $\lim_{x \rightarrow \infty} \left(\frac{x + \sqrt{x}}{x - \sqrt{x}} \right)^{2x}$;
9. $\lim_{x \rightarrow 0} \left(\frac{4^x + 9^x}{2} \right)^{\frac{1}{x}}$; 10. $\lim_{x \rightarrow 0} (1 + x^2 e^x)^{\frac{1}{1-\cos 2x}}$; 11. $\lim_{x \rightarrow 3} (x-2)^{\frac{1}{x-3}}$; 12. $\lim_{x \rightarrow 0} (1 + x^2)^{ctg^2 x}$;
- 13*. $\lim_{x \rightarrow 0} \left(\frac{\cos x}{\cos 2x} \right)^{\frac{1}{x^2}}$; 14*. $\lim_{x \rightarrow 0} \ln(1 + x + x^2 + \dots + x^n)^{\frac{1}{nx}}$; 15*. $\lim_{x \rightarrow 0} (e^{x^2} + x^2)^{\frac{1}{1-\cos x}}$;

Răspunsuri: 1. e^4 ; 2. $e^{-4} = \frac{1}{e^4}$; 3. $e^{\frac{1}{2}} = \sqrt{e}$; 4. e ; 5. $e^{\frac{1}{2\pi}}$; 6. $e^{\frac{4}{3}} = \sqrt[3]{e^4}$; 7. ∞ ; 8. ∞ ; 9. 6; 10. $e^{\frac{1}{2}} = \sqrt{e}$;
 11. e 12. e. 13. e^2 ; 14. $\frac{1}{n}$; 15. e^4 ;

CALCULUL LIMITELOR DE FUNCȚII

Elev:.....

PROF. STAN ADRIAN

CAZUL DE NEDETERMINARE $\frac{0}{0}$

Grup Școlar „Costin Nenitescu”, Buzău

Exerciții rezolvate:

- $\lim_{x \rightarrow -1} \frac{x^2 - 1}{x^2 + 3x + 2} = \lim_{x \rightarrow -1} \frac{(x-1)(x+1)}{(x+1)(x+2)} = \lim_{x \rightarrow -1} \frac{(x-1)}{(x+2)} = -2$;
- $\lim_{x \rightarrow 0} \frac{\sin 12x}{4x} = \lim_{x \rightarrow 0} \frac{\sin 12x}{12x} \cdot 3 = 3$;
- $\lim_{x \rightarrow 0} \frac{\sin 5x - \sin 3x}{\sin x} = \lim_{x \rightarrow 0} \frac{\sin 5x}{5x} \cdot \frac{x}{\sin x} \cdot 5 - \frac{\sin 3x}{3x} \cdot \frac{x}{\sin x} \cdot 3 = 5 - 3 = 2$;
- $\lim_{x \rightarrow 0} \frac{e^{3x} - 1}{e^{5x} - 1} = \lim_{x \rightarrow 0} \frac{e^{3x} - 1}{3x} \cdot \frac{5x}{e^{5x} - 1} \cdot \frac{3}{5} = \frac{3}{5}$;
- $\lim_{x \rightarrow 0} \frac{\ln(1 + \arcsin x)}{\sin 3x} = \lim_{x \rightarrow 0} \frac{\ln(1 + \arcsin x)}{\arcsin x} \cdot \frac{\arcsin x}{x} \cdot \frac{3x}{\sin 3x} \cdot \frac{1}{3} = \frac{1}{3}$
- $\lim_{x \rightarrow 5} \frac{\sqrt{x-1} - \sqrt{2x-6}}{x-5} = \lim_{x \rightarrow 5} \frac{x-1-2x+6}{(x-5) \cdot (\sqrt{x-1} + \sqrt{2x-6})} =$
 $\lim_{x \rightarrow 5} \frac{-x+5}{(x-5) \cdot (\sqrt{x-1} + \sqrt{2x-6})} = \frac{-1}{4}$;
- $\lim_{x \rightarrow 0} \frac{\sqrt[3]{x+8} - 2}{x^2 + 4x} = \lim_{x \rightarrow 0} \frac{x+8-8}{x(x+4)(\sqrt[3]{(x+8)^2} + 2\sqrt[3]{x+8} + 4)} = \frac{1}{48}$;
- $\lim_{x \rightarrow a} \frac{\sqrt[n]{x} - \sqrt[n]{a}}{x-a} = \lim_{x \rightarrow a} \frac{\sqrt[n]{x} - \sqrt[n]{a}}{\sqrt[n]{x^n} - \sqrt[n]{a^n}} = \lim_{x \rightarrow a} \frac{1}{\sqrt[n]{x^{n-1}} + \sqrt[n]{x^{n-2}} \cdot a + \dots + \sqrt[n]{x \cdot a^{n-2}} + \sqrt[n]{a^{n-1}}} = \frac{\sqrt[n]{a}}{a \cdot n}$

Rezolvă singur exercițiile:

Exerciții propuse:

- $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 - 5x + 6}$;
- $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^3 - 8}$;
- $\lim_{x \rightarrow 0} \frac{\sin 5x}{\sin 6x}$;
- $\lim_{x \rightarrow 2} \frac{\sin(x^2 - 6x + 8)}{\sin(x^2 - 5x + 6)}$;
- $\lim_{x \rightarrow 3} \frac{e^{x-3} - 1}{x^3 - 27}$;
- $\lim_{x \rightarrow 0} \frac{8^x + 7^x - 6^x - 5^x}{4^x - 3^x}$
- $\lim_{x \rightarrow 0} \frac{\ln(1 + \operatorname{tg} 3x)}{4x}$;
- $\lim_{x \rightarrow 4} \frac{\sqrt{x+5} - 3}{\sqrt{5-x} - 1}$;
- $\lim_{x \rightarrow 0} \frac{\sin 3x + \sin 4x}{\sin 5x + \sin 6x}$
- $\lim_{x \rightarrow 1} \frac{3^{x+2} - 27}{x^2 - x}$;
- $\lim_{x \rightarrow 2} \frac{2^x - x^2}{x^2 - x - 2}$;
- $\lim_{x \rightarrow 0} \frac{9^{\operatorname{tg} 15x} - 1}{30x}$;

Temă:

- $\lim_{x \rightarrow 0} \frac{x^5 + x^3}{x^3}$; 2. $\lim_{x \rightarrow 0} \frac{x^4 + 8x}{x^3 - 4x}$; 3. $\lim_{x \rightarrow 0} \frac{\sin^2 4x}{8x^2}$; 4. $\lim_{x \rightarrow 0} \frac{\operatorname{tg} 6x}{\operatorname{tg} 3x}$; 5. $\lim_{x \rightarrow 0} \frac{\sin(2x \sin 4x)}{2x^2}$;
- $\lim_{x \rightarrow 1} \frac{x^2 - 4x + 3}{x^3 - 2x^2 + 2x - 1}$; 7. $\lim_{x \rightarrow 0} \frac{e^{5x} - 1}{e^{8x} - 1}$; 8. $\lim_{x \rightarrow 0} \frac{e^{\sin 3x} - 1}{6x}$; 9. $\lim_{x \rightarrow 2} \frac{5^x - 25}{x^2 - 3x + 2}$; 10. $\lim_{x \rightarrow 1} \frac{2x - 2}{\sqrt{10 - x} - 3}$
- $\lim_{x \rightarrow 0} \frac{\ln(1 + \sin 12x)}{\ln(1 + \sin 6x)}$; 12. $\lim_{x \rightarrow 0} \frac{\sqrt[3]{x+27} - 3}{x}$; 13. $\lim_{x \rightarrow 3} \frac{3^x - x^3}{x^2 - 4x + 3}$; 14. $\lim_{x \rightarrow 0} \frac{e^{\operatorname{tg} x} - e^{\sin x}}{x^3}$;
- $\lim_{x \rightarrow 1} \frac{x + x^2 + \dots + x^n - n}{x - 1}$; 16. $\lim_{x \rightarrow 1} \frac{x^7 - 7(x-1) - 1}{(x-1)^2}$; 17. $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$;

Răspunsuri: 1. 1; 2. -2; 3. 2; 4. $\frac{1}{2}$; 5. 4; 6. -3; 7. $\frac{5}{8}$; 8. $\frac{1}{2}$; 9. $25\ln 5$; 10. -12; 11. 2; 12. $\frac{1}{27}$; 13. $9-27\ln 3$; 14. $\frac{1}{2}$;
15. $\frac{(n-1)n}{2}$; 16. 21; 17. $\frac{1}{2}$;