

## Inegalitati privind numarul 2008

-Marcu Stefan Florin –profesor Calarasi

## PROBLEMA PROPUSA

Sa se demonstreze inegalitatile :

a)  $1004 \cdot \sqrt[2008]{2008} < 1003 + \sqrt{2008}$

b)  $\sqrt{2008} > (1 + \frac{2}{5})(1 + \frac{2}{9}) \dots (1 + \frac{2}{8029})$ .

SOLUTIE :

a) Aplicam inegalitatea mediilor numerelor :  $\underbrace{1,1,1,\dots,1}_{2006\text{-ori}}, \sqrt{2008}, \sqrt{2008}$  si

obtinem :  $\sqrt[2008]{2008} < \frac{1+1+\dots+1+2\sqrt{2008}}{2008} = \frac{2006+2\sqrt{2008}}{2008} = \frac{1003+\sqrt{2008}}{1004}$ ,

de unde se obtine inegalitatea a) .

b) Vom demonstra in prealabil inegalitatea :  $\sqrt{1+\frac{1}{n}} > 1 + \frac{2}{4n+1}$ , care areloc  $\forall n > 0, n \in \mathbb{N}$ .

Ridicand la patrat relatia de mai sus obtinem :

$$1 + \frac{1}{n} > 1 + \frac{4}{4n+1} + \frac{4}{(4n+1)^2} \Leftrightarrow (4n+1)^2 > 4n(4n+1) + 4n$$

$$\Leftrightarrow 16n^2 + 8n + 1 > 16n^2 + 8n \text{ (adevarat)}.$$

Dam lui  $n$ , valorile  $1, 2, \dots, 2007$  in inegalitatea de mai sus si inmultim inegalitatile obtinute, astfel :

$$\sqrt{\frac{2}{1}} > 1 + \frac{2}{5}, \sqrt{\frac{3}{2}} > 1 + \frac{2}{9}, \dots, \sqrt{\frac{2008}{2007}} > 1 + \frac{2}{8029}$$
. Prin inmultirea

radicalilor si simplificarea numerelor  $2, 3, \dots, 2007$  se obtine inegalitatea din enunt.

## PROBLEMA PROPUSA

-MARCU STEFAN FLORIN –PROFESOR CALARASI

Sa se gaseasca toate tripletele de numere naturale pare , nenule , de forma  $(a,b,c)$  , cu  $a \leq b \leq c$  si  $a^3 + b^3 + c^3 = 2008$ .

SOLUTIE :

Observam ca  $2008 = 8 \cdot 251$

Atunci , ecuatia din enunt poate fi redusa la ecuatia :

$$x^3 + y^3 + z^3 = 251 , \text{ unde } x = \frac{a}{2} , y = \frac{b}{2} , z = \frac{c}{2} \text{ cu } x, y, z \in \mathbb{N}^* \text{ si } x \leq y \leq z .$$

Vom analiza cazurile posibile .

Observam ca  $x^3 \leq 251$  ,  $y^3 \leq 251$  ,  $z^3 \leq 251$  , deci  $1 \leq x, y, z < 7$  .

De asemeni avem inegalitatile :

$$3x^3 \leq x^3 + y^3 + z^3 \leq 3z^3 \Leftrightarrow 3x^3 \leq 251 \leq 3z^3 .$$

$$\text{Deci } x^3 \leq \frac{251}{3} \Rightarrow x < 5 \text{ si } z^3 \geq \frac{251}{3} \Rightarrow z > 4 . \text{ Dar } z < 7 \text{ deci } z \in \{5, 6\} .$$

Daca  $z=5$  , obtinem  $x^3 + y^3 = 251 - 125 = 126$ .

Dar  $x \in \{1, 2, 3, 4\}$  si analizand situatiile posibile , obtinem  $x=1$  si  $y=5$ .

Deci  $a=2$ ,  $b=10$  ,  $c=10$  .

Daca  $z=6$  , obtinem  $x^3 + y^3 = 251 - 216 = 35$ .

Se va obtine  $x=2$  si  $y=3$  .

Deci  $a=4$  ,  $b=6$  ,  $c=12$  .

Asadar , tripletele cautate sunt  $(2, 10, 10)$  si  $(4, 6, 12)$  .

## Problema propusa

-Marcu Stefan Florin-profesor Calarasi

Sa se demonstreze inegalitatea :

$$\sqrt[3]{2008} < \frac{4}{3} + \frac{1}{3\sqrt[3]{4}} + \frac{1}{3\sqrt[3]{9}} + \dots + \frac{1}{3\sqrt[3]{2007^2}} .$$

SOLUTIE :

Consideram functia  $f : (0, +\infty) \rightarrow \mathbb{R}$ ,  $f(x) = \sqrt[3]{x}$ . Avem  $f'(x) = \frac{1}{3} x^{-\frac{2}{3}}$  si

$f''(x) = -\frac{2}{9} x^{-\frac{5}{3}} < 0$ , deci functia  $f$  este concava.

Aplicand Teorema lui Lagrange, functiei  $f$  pe un interval de forma

$[k, k+1]$ ,  $k \in \mathbb{N}^*$ , obtinem ca  $(\exists) c_k \in (k, k+1)$  astfel incat  $f(k+1) -$

$f(k) = f'(c_k)$ . Dar  $f'$  este strict descrescatoare pe  $(0, +\infty)$ , deoarece este concava. Deci :

$$f'(k+1) < f(k+1) - f(k) < f'(k) \Leftrightarrow \frac{1}{3} (k+1)^{-\frac{2}{3}} < \sqrt[3]{k+1} - \sqrt[3]{k} < \frac{1}{3} k^{-\frac{2}{3}} \Leftrightarrow$$

$$\frac{1}{\sqrt[3]{(k+1)^2}} < 3(\sqrt[3]{k+1} - \sqrt[3]{k}) < \frac{1}{\sqrt[3]{k^2}} .$$

Dam valori lui  $k$  de la 1 la 2007, in relatia de mai sus, si apoi adunam

relatiile. Obtinem :

$$k=1: \quad \frac{1}{\sqrt[3]{4}} < 3(\sqrt[3]{2} - 1) < 1$$

$$k=2: \quad \frac{1}{\sqrt[3]{9}} < 3(\sqrt[3]{3} - \sqrt[3]{2}) < \frac{1}{\sqrt[3]{4}}$$

$$k=2007: \quad \frac{1}{\sqrt[3]{2008^2}} < 3(\sqrt[3]{2008} - \sqrt[3]{2007}) < \frac{1}{\sqrt[3]{2007^2}}$$

Prin adunare obtinem :

$$3(\sqrt[3]{2008} - 1) < 1 + \frac{1}{\sqrt[3]{4}} + \dots + \frac{1}{\sqrt[3]{2007^2}} \Leftrightarrow$$

$$\sqrt[3]{2008} < \frac{4}{3} + \frac{1}{\sqrt[3]{4}} + \frac{1}{\sqrt[3]{9}} + \dots + \frac{1}{\sqrt[3]{2007^2}} .$$