A new proof of Bertrand's postulate
(the simplest possible demonstration of Bertrand's postulate)

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Abstract

In this note, we give an elementary proof of Bertrand’s postulate.We make use of a theorem proved in 2006 by mathematician M. El Bachraoni .

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1 Introduction

It is known that the ring Z of integers numbers is factorial ring and a complete system of irreducible elements is the set of the prime numbers. The prime numbers in mathematics is evidenced by the fundamental theorem of arithmetics.

Interesting is the prime numbers among the natural numbers and the fact that problems of this nature are made very simple but it demonstrates very difficult. This problem was made by Polish mathematician W. Sierpinski: For all natural numbers $n > 1$and $k \leq n$ there is at least one prime in the interval $[kn,(k+1)n]$.

The case $k = 1$ was filed in 1845 by French mathematician J. Bertrand and proved by Russian mathematician PL Cebâşev. A simplified demonstration gave a mathematician P. Erdos in 1932 and a recent Romanian mathematician in M. Tena ([3]).The case $k = 2$ was proved in 2006 by mathematician M. El Bachraoni (see [1]). Demonstration given by M. Bachraoni It is relatively short and not too complicated. This may be free on the internet at [4].

In this note we presents a refinement of Bertrand’s postulate and we give the simplest demonstration of this postulate.

2 Main Result

Theorem 1. *For any positive integer* $n > 1$ *there is a prime number between* $2n$

and $3n$. *(see the proof in* [1] *or* [4]*).*

Demonstration that is typical for many theorems of number theory and is based on multiple inequality valid for large values of n which can be calculated effectively. For the rest of n values are all kinds of improvisation, basic, but difficult to follow.

Theorem 2. *Theorem 1.* $\Longrightarrow n < p < \frac{3(n+1)}{2}, \forall\, n \geq 1$.*(A refinement of Bertrand' postulate – see also the proof in* [1] *or* [4] *).*

Proof.

If $n = 1$, we have $1 < p < 3$, true. If $n = 2$, $2 < p < \frac{9}{2}$, true.

For , $n = even$, $n = 2k$, we have $n = 2k < p < 3k < \frac{3(2k+1)}{2} = \frac{3(n+1)}{2}$.

For, $n = odd$, $n = 2k + 1$, we have :

$n = 2k + 1 < 2k + 2 = 2(k+1) < p < 3(k+1) = \frac{3(n+1)}{2}$. □

Theorem 3. *Theorem 2.* $\Longrightarrow$ *Bertrand' postulate.*

Proof.

We have $n < p < \frac{3(n+1)}{2}$. Because $\frac{3(n+1)}{2} < 2n \Leftrightarrow n > 3$, results, $n < p < 2n, \forall n > 3$. □

Theorem 4. *Theorem 1.* $\Longrightarrow$ *Bertrand' postulate.*

Proof.

Case I.

$n = even = 2k < p < 3k < 4k = 2n.$

Case II.

$n = odd = 2k + 1 < 2k + 2 = 2(k+1) < p < 3(k+1) < 4k + 2 = 2(2k+1) = 2n.$

This is the simplest demonstration of *Bertrand' postulate.*

The conjecture about prime numbers distribution among natural numbers, is likely to be resolved, because recently (2008) is proved a formula that generates prime numbers. The result is due mathematician Rafael Jakimczuk and demonstrate in [2] (also may be free study to [5].).

This formula is:

$$p_n = n\log n + \log(n\log n)\left(n - Li(n\log n)\right) +$$
$$+ \sum_{k=2}^{\infty} \frac{(-1)^k Q_{k-1}(\log(n\log n))}{k!n^{k-1}\log^{k-1} n}\left(n - Li(n\log n)\right)^k + O(h(n)),\ n \geq 4.$$

Where, $Li(x) = \int_2^x \frac{1}{\log t}dt, h(n) = \frac{n\log^2 n}{\exp\left(d\sqrt{\log n}\right)}$, $Q_{k-1}(x)$ are polynomials and $O(h(n))$ is better mistake term in others previous known aproximate formulas for p_n.

References

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