CATEVA PROBLEME DEOSEBITE CU INECUATII

1. Demonstrati ca:

a) $\sqrt{n}(\sqrt{n+1}-\sqrt{n}) < \frac{1}{2}, \forall n \in N^*;$ (relatie cunoscuta din manualul de cls a X a)

b)

$$\frac{\sqrt{1\cdot 2}}{1}+\frac{\sqrt{2\cdot 3}}{2}+\frac{\sqrt{3\cdot 4}}{3}+...+\frac{\sqrt{2008\cdot 2009}}{2008} > 2008$$

Prof. Dobre Andrei (pentru cls. a VIII a si/sau a X a)

2. Demonstrati ca $a^2+b^2+c^2 \geq \frac{2}{3}$(bc-ab-ac) $\forall a,b,c \in R$

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EZOLVARI SUBIECTE

$$1)a)\sqrt{n+1}-\sqrt{n}=\frac{1}{\sqrt{n+1}+\sqrt{n}}<\frac{1}{2\sqrt{n}} \Rightarrow \sqrt{n}\cdot\left(\sqrt{n+1}-\sqrt{n}\right)<\frac{1}{2}$$

$$b)\sqrt{n}\cdot\left(\sqrt{n+1}-\sqrt{n}\right)<\frac{1}{2}$$

$$2\left(\sqrt{n+1}-\sqrt{n}\right)<\frac{1}{\sqrt{n}}$$

$$2(n+1-n)<\frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n}}$$

$$2<\frac{\sqrt{n+1}}{\sqrt{n}}+1 \Rightarrow 2<\frac{\sqrt{n(n+1)}}{n}+1$$

$n=1$

$n=2$

....

$n=2008$

$$2\cdot 2008<2008+\frac{\sqrt{1\cdot 2}}{1}+\frac{\sqrt{2\cdot 3}}{2}+\cdots\cdots\cdot\frac{\sqrt{2008\cdot 2009}}{2008}$$

$$\frac{\sqrt{1\cdot 2}}{1}+\frac{\sqrt{2\cdot 3}}{2}+\cdots\cdots\cdots+\frac{\sqrt{2008\cdot 2009}}{2008}>2008$$

$$2)(a+b+c)^2+(a-b+c)^2+(a+b-c)^2\geq 0$$

$$a^2+b^2+c^2+2ab+2ac+2bc+$$

$$a^2+b^2+c^2-2ab+2ac-2bc+$$

$$a^2+b^2+c^2+2ab-2ac-2bc$$

$$3a^2+3b^2+3c^2+2ab-2bc+2ac\geq 0$$

$$3\left(a^2+b^2+c^2\right)\geq 2(bc-ab-ac)$$

$$a^2+b^2+c^2\geq\frac{2}{3}(bc-ab-ac)$$