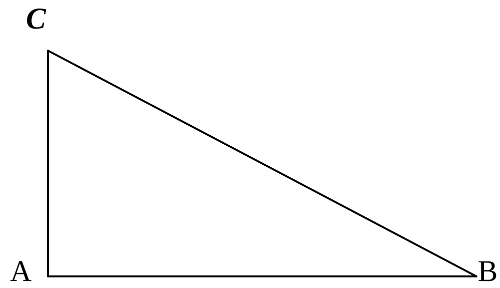
Formule trigonometrice



Sinus şi Cosinus

$\sin B = \frac{b}{a}$

$\cos B = \frac{c}{a}$

$tgB = \frac{b}{a}$

$ctgB = \frac{c}{b}$

1) $\sin^2 B + \cos^2 B = \left(\frac{b}{a}\right)^2 + \left(\frac{c}{a}\right)^2 = \frac{b^2 + c^2}{a^2} = \frac{a^2}{c^2} = 1$

Formula fundamentală a trigonometriei. $\sin^2 B + \cos^2 B = 1, (\forall) B \in \left(0, \frac{\pi}{2}\right)$

2) $\begin{cases} \frac{\sin B}{\cos B} = \frac{b}{a} \times \frac{a}{c} = \frac{b}{c} = tgB \Rightarrow tgB = \frac{\sin B}{\cos B}, (\forall) B \in \left(0, \frac{\pi}{2}\right) \\ \frac{\cos B}{\sin B} = ctgB \Rightarrow ctgB = \frac{\cos B}{\sin B}, (\forall) B \in \left(0, \frac{\pi}{2}\right) \\ tgB \times ctgB = 1 \end{cases}$

3) $\begin{cases} 1 + tg^2 B = 1 + \frac{b^2}{c^2} = \frac{c^2 + b^2}{c^2} = \frac{a^2}{c^2} = \frac{1}{\cos^2 B} \\ 1 + tg^2 B = \frac{1}{\cos^2 B} \\ 1 + ctg^2 B = \frac{1}{\sin^2 B} \end{cases}$

$C = \frac{\pi}{2} - B$

4) $\begin{cases} \sin B = \cos\left(\frac{\pi}{2} - B\right) \\ \cos B = \sin\left(\frac{\pi}{2} - B\right) \\ tgB = ctg\left(\frac{\pi}{2} - B\right) \\ ctgB = tg\left(\frac{\pi}{2} - B\right) \end{cases}$

5)

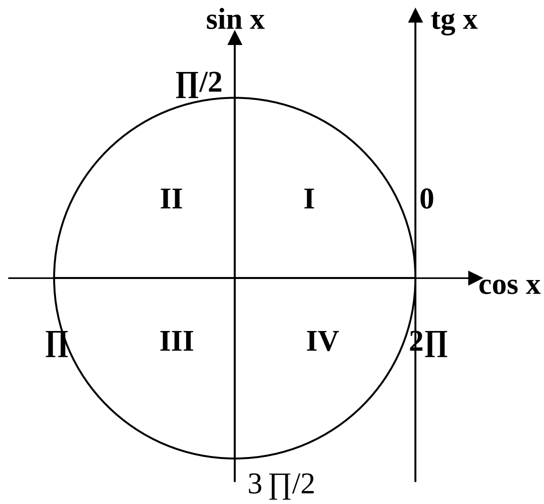
x	0		$\frac{\pi}{2}$		π		$\frac{3\pi}{2}$		2π
sin x	**0**	+++	1	+++	0	- - -	1	- - -	0
cos x	**1**	+++	0	- - -	1	- - -	0	+++	1

6) x şi x+2kπ , k $\in$ Z au aceeaşi extremitate

$$\sin(x+2k\pi)=\sin x$$
$$\cos(x+2k\pi)=\cos x$$
$$k=-1$$

6')

$$\sin(x-2\pi)=\sin x$$
$$\cos(x-2\pi)=\cos x$$



7) Cadranul II

$$\sin(\pi-x)=\sin x$$
$$\cos(\pi-x)=-\cos x$$
$$\cos x=-\cos(\pi-x)$$

8)Cadranul III

$$\sin(\pi+x)=-\sin x$$
$$\cos(\pi+x)=-\cos x$$

9)Cadranul IV

$$\sin(2\pi-x)=-\sin x$$
$$\cos(2\pi-x)=-\cos x$$

10)

$$\sin(-x)=-\sin x$$
$$\cos(-x)=\cos x$$

11)

$$\sin(x+2\pi)=\sin x$$
$$\cos(x+2\pi)=\cos x$$

Tangent

x	0		$\frac{\pi}{2}$		π		$\frac{3\pi}{2}$		2 π
tg x	**0**	+++	|	- - -	0	+++	|	- - -	0

x ≠ $\frac{\pi}{2}$

în general (∃) tg x

x ≠ $\frac{3\pi}{2}$ pt x ≠ (2k+1) $\frac{\pi}{2}$; k $\in$ Z

Def. yt = tgx

11) $$\begin{cases} tg(x+\pi)=tgx \\ tg(\pi-x)=-tgx \\ tg(x+k\pi)=tgx, k\in Z \end{cases}$$

12) $tg(-x)=-tgx$

13) $tgx=\dfrac{\sin x}{\cos x}$

14) $1+tg^2x=\frac{1}{\cos^2x}$

Cotangent

x	0		$\frac{\pi}{2}$		π		$\frac{3\pi}{2}$		2 π
ctg x	|	+++	0	- - -	|	+++	0	- - -	|

15) $\begin{cases} ctg(x+\pi)=ctgx \\ ctg(x-\pi)=ctgx \\ ctg(x+k\pi)=ctgx, (\forall)k\in Z \end{cases}$

16) $ctg(-x)=-ctgx$

17) $ctgx=\frac{\cos x}{\sin x}$

18) $tgx\times ctgx=1$

19) $1+ctg^2x=\frac{1}{\sin^2x}$

20) $\begin{cases} \cos\left(\frac{\pi}{2}-x\right)=\sin x \\ \sin\left(\frac{\pi}{2}-x\right)=\cos x \\ tg\left(\frac{\pi}{2}-x\right)=ctgx \\ ctg\left(\frac{\pi}{2}-x\right)=tgx \end{cases}$

21) $\cos(x-y)=cox\cos y+\sin x\sin y, (\forall)x,y\in R$

22) $\cos(x+y)=\cos x\cos y-\sin x\sin y$

23) $\sin(x+y)=\sin x\cos y+\cos x\sin y$

24) $\sin(x-y)=\sin x\cos y-\cos x\sin y$

25) $\cos 2x=\cos^2x-\sin^2x$ forma omogenă

25') $\cos 2x=2\cos^2x-1$ numai în funcţie de cos α

25'') $\cos 2x=1-2\sin^2x$ numai în funcţie de sin α

$(\forall)x\in R$

26) $\sin 2x=2\sin x\cos x$

27) $1+\cos 2x=2\cos^2x\rightarrow\cos^2x=\frac{1+\cos 2x}{2}$

28) $1-\cos 2x=2\sin^2x\rightarrow\sin^2x=\frac{1-\cos 2x}{2}$

29) $tg(x+y)=\frac{tgx+tgy}{1-tgxtgy}$

30) $tg(x-y)=\frac{tgx-tgy}{1+tgxtgy}$

31) $tg2x = \frac{2tgx}{1 - tg^2 x}$

32) $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$

33) $\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$

34) $\cos a \cos b = \frac{\cos(a-b) + \cos(a+b)}{2}$

35) $\sin a \sin b = \frac{\cos(a-b) - \cos(a+b)}{2}$

36) $\sin a \cos b = \frac{\sin(a-b) + \sin(a+b)}{2}$

Dacă $tg\frac{x}{2} = t$

37) $\sin x = \frac{2t}{1+t^2}$

38) $\mathbf{cos}\, x = \frac{1-t^2}{1+t^2}$

39) $tgx = \frac{2t}{1-t^2}$

40) $tg\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha}$

41) $\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}, (\forall)\alpha,\beta \in R$

42) $\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}, (\forall)\alpha,\beta \in R$

43) $\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}, (\forall)\alpha,\beta \in R$

44) $\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}, (\forall)\alpha,\beta \in R$

A+B+C=π

45) $\sin A + \sin B + \sin C = 4\cos\frac{C}{2}\cos\frac{A}{2}\cos\frac{B}{2}$

46) $\mathbf{cos}\, A + \mathbf{cos}\, B + \mathbf{cos}\, C - 1 = 4\,\mathbf{sin}\frac{C}{2}\,\mathbf{sin}\frac{A}{2}\,\mathbf{sin}\frac{B}{2}$

Ecuatii trigonometrice

$\mathbf{sin}\, x = a \Rightarrow x \in \{(-1)^k \mathbf{arcsin}\, a + k\prod\} k \in Z, a \in [-1,1]$

$\mathbf{cos}\, x = a \Rightarrow x \in \{+- \mathbf{arccos}\, a + 2k\prod\} k \in Z, a \in [-1,1]$

$tgx = a \Rightarrow x \in \{arctga + k\prod\} k \in Z, a \in R$

$ctgx = a \Rightarrow x \in \{arctgx + k\prod\} k \in Z, a \in R$