**CONCURSUL DE MATEMATICĂ APLICATĂ „ADOLF HAIMOVICI”**

Etapa locală – Constanța, 15.02.2015

**Clasa a X-a**

filiera teoretică: profil real, specializarea ştiinţe ale naturii

**Barem de corectare și notare**

**SUBIECTUL 1.**

$x^2+y^2+x+y-2=x(x+1)+y(y+1)-2=$nr. par ..... **2p.**

$\Rightarrow 2^{|x-y|}+1$ par pentru $2^{|x-y|}=1$ .....**2p**

$|x-y|=0\Rightarrow x=y$ .....**1p**

$x(x+1)=2\Rightarrow x\in\{-2,1\}\Rightarrow (x,y)\in\{(-2,-2),(1,1)\}$ .....**2p**

**SUBIECTUL 2.**

**a)** $S=\frac{1}{\log_2 n!}+\frac{1}{\log_3 n!}+...+\frac{1}{\log_n n!}$ .....**2p**

$S=\log_{n!}2+\log_{n!}3+...+\log_{n!}n$ .....**1p**

$S=\log_{n!}2\cdot 3\cdot...\cdot n=\log_{n!}n!=1$ .....**1p**

**b)** Fie $p=\left(\frac{bc}{a}\right)^{\lg\frac{b}{c}}\cdot\left(\frac{ac}{b}\right)^{\lg\frac{c}{a}}\cdot\left(\frac{ab}{c}\right)^{\lg\frac{a}{b}}=a^{\lg\frac{c}{a}+\lg\frac{a}{b}-\lg\frac{b}{c}}b^{-\lg\frac{c}{a}+\lg\frac{a}{b}+\lg\frac{b}{c}}c^{\lg\frac{c}{a}-\lg\frac{a}{b}+\lg\frac{b}{c}}$ .....**1p**

$p=a^{2\lg\frac{c}{b}}b^{2\lg\frac{a}{c}}c^{2\lg\frac{b}{a}}$ .....**1p**

$\lg p=0\Rightarrow p=1\in\mathbf{N}$ .....**1p**

**SUBIECTUL 3.**

**a)** $x_1=1+i,\quad x_2=1-i$ .....**1p**

$x_1^{2015}+x_2^{2015}=x_1\cdot x_1^{2014}+x_2\cdot x_2^{2014}=(1+i)\left((1+i)^2\right)^{1007}+(1-i)\left((1-i)^2\right)^{1007}=2^{2008}$ .....**3p**

**b)** $x_1^5+x_2^5=-8$ .....**1p**

$x_1^3+x_2^3=-4$ .....**1p**

$x_1^5+x_2^5=2(x_1^3+x_2^3)$ .....**1p**

**SUBIECTUL 4.**

**a)** $|z_1+z_2|\le|z_1|+|z_2|$ .....**1p**

$|z_1+z_2+z_3|\le|z_1+z_2|+|z_3|\le|z_1|+|z_2|+|z_3|$ .....**2p**

**b)** $z^3+1-z\left(z^2+1\right)+z+1=2$ .....**2p**

$2=\left|z^3+1-z\left(z^2+1\right)+z+1\right|\le\left|z^3+1\right|+\left|z(z^2+1)\right|+|z+1|=$ .....**1p**

$\left|z^3+1\right|+|z|\left|z^2+1\right|+|z+1|=\left|z^3+1\right|+\left|z^2+1\right|+|z+1|$ .....**1p**