OLIMPIADA DE MATEMATICĂ

Etapa locală – Constanța, 15.02.2015

Clasa a VI-a

Barem de corectare şi notare

Subiectul 1.

$5|145b$, $5|15c$, $5|2015 \Rightarrow 5|27a$, $5\nmid 27 \Rightarrow 5|a$, a prim $\Rightarrow a=5$ ……………..……….......... ***2p***

$29b+3c=376 \Rightarrow b<13$, b prim $\Rightarrow b\in\{2,3,5,7,11\}$ ………...…………………..….…….. ***2p***

Verificarea că pentru $b\in\{2,3,5,7\}$ nu se obţin soluţii convenabile ……...……………...……. ***2p***

$b=11 \Rightarrow c=19$. Soluţie finală: $a=5$, $b=11$, $c=19$ ***1p***

Subiectul 2.

a) $\overline{abcd}+\dfrac{\overline{abcd}}{6}+\dfrac{\overline{abcd}}{6^2}+...+\dfrac{\overline{abcd}}{6^n}=\overline{abcd}\cdot\left(1+\dfrac{1}{6}++\dfrac{1}{6^2}+...+\dfrac{1}{6^n}\right)$ ……..……............***1p***

$1+\dfrac{1}{6}+\dfrac{1}{6^2}+...+\dfrac{1}{6^n}=\dfrac{6^{n+1}-1}{5\cdot 6^n}$ …………………………………………..…......…......***1p***

$\dfrac{\overline{abcd}}{6^n}=1 \Rightarrow \overline{abcd}=6^n$ …………………………………………………………….***1p***

$n\in\{4;5\}$, $\overline{abcd}\in\{1296;7776\}$ ……………………………………..………...….....…***1p***

b) $2016^{2015}=(2015+1)^{2015}=M_{2015}+1^{2015}=M_{2015}+1$

$2014^{2015}=(2015-1)^{2015}=M_{2015}-1^{2015}=M_{2015}-1$ …………………………………........***1p***

$A=M_{2015}+1+M_{2015}-1=M_{2015}=2015\cdot k$, $k\in\mathbf{N}^*$……………………………..….***1p***

$2015=5\cdot 13\cdot 31 \Rightarrow A=5\cdot 13\cdot 31\cdot k$, $k\in\mathbf{N}^*$, deci A are cel puţin 3 divizori numere prime.***1p***

Subiectul 3.

a) $OA+AB=OB=2^{x+1}$ (sau $OB=2\cdot OA$, $OC=4\cdot OA$) ***1p***

$BC=OC-OB=2^{x+2}-2^{x+1}=2^{x+1}=OA+AB$ ($BC=OB=OA+AB$) ***1p***

a) $MN=2^{x+1}+2^{x-1}$ (sau $MN=\dfrac{5}{2}\cdot OA$) ***2p***

$x=3\,\text{cm}$ ***2p***

$AC=24\,\text{cm}$ ***1p***

Subiectul 4.

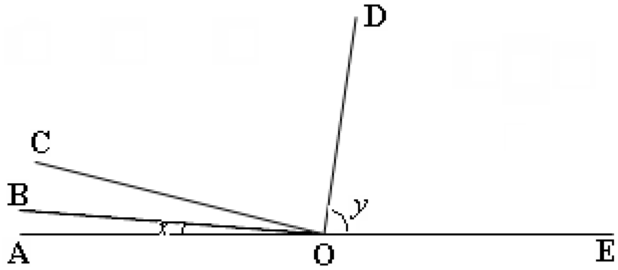
Notăm $m(\widehat{AOB})=x$, $m(\widehat{DOE})=y \Rightarrow x+\dfrac{3}{2}x+\dfrac{15}{2}x+y=180^0$ .... ***2 p***

$\dfrac{15}{2}x<90^0 \Rightarrow x<12^0$ *(1)*..***1 p***

$y=180^0-10x<90^0 \Rightarrow x>9^0$ *(2)*..***2 p***

$(1)+(2) \Rightarrow m(\widehat{AOB})\in\{10^0;11^0\}$, $m(\widehat{BOC})\in\{15^0;16^0 30'\}$, $m(\widehat{COD})\in\{75^0;82^0 30'\}$, $m(\widehat{DOE})\in\{80^0;70^0\}$
....***1 p***

Măsurile sunt numere naturale $\Rightarrow m(\widehat{AOB})=10^0$, $m(\widehat{BOC})=15^0$, $m(\widehat{COD})=75^0$, $m(\widehat{DOE})=80^0$ ... ***1 p***



Notă : **Orice altă soluţie corectă, diferită de cea din barem, va primi punctaj maxim .**