CONCURSUL DE MATEMATICĂ APLICATĂ „ADOLF HAIMOVICI”

ETAPA LOCALĂ

28 februarie 2015

BAREM

CLASA A XI-A

1.)	**Din oficiu**	**1p**
a)	$\det A=\begin{vmatrix}0 & -1\\ 1 & 0\end{vmatrix}=1$, $\det B=\begin{vmatrix}2 & 3\\ 3 & 4\end{vmatrix}=-1$	**2p**
b)	$\det A=1$, $\det B=-1$ deci cele două matrici sunt inversabile	**1p**
	$A^{-1}=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$, $B^{-1}=\begin{pmatrix}-4 & 3\\ 3 & -2\end{pmatrix}$	**3p**
	$X=A^{-1}B^{-1}$	**1p**
	$X=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\begin{pmatrix}-4 & 3\\ 3 & -2\end{pmatrix}=\begin{pmatrix}3 & -2\\ 4 & -3\end{pmatrix}$	**2p**

2.)	**Din oficiu**	**1p**
a)	Notând cu r raţia progresiei aritmetice obţinem: $a_2=a_1+r \Rightarrow r=a_2-a_1$ $a_3=a_1+2r \Rightarrow 2r=a_3-a_1$ etc.	**1p**
	$\Delta=\begin{vmatrix}a_1 & a_2 & a_3\\ a_4 & a_5 & a_6\\ a_7 & a_8 & a_9\end{vmatrix}=\begin{vmatrix}a_1 & a_2-a_1 & a_3-a_1\\ a_4 & a_5-a_4 & a_6-a_4\\ a_7 & a_8-a_7 & a_9-a_7\end{vmatrix}=\begin{vmatrix}a_1 & r & 2r\\ a_4 & r & 2r\\ a_7 & r & 2r\end{vmatrix}=0$	**4p**
b)	Notând cu q raţia progresiei geometrice obţinem: $a_2=a_1q$, $a_3=a_1q^2$ etc.	**1p**
	$\Delta=\begin{vmatrix}a_1 & a_1q & a_1q^2\\ a_1q^3 & a_1q^4 & a_1q^5\\ a_1q^6 & a_1q^7 & a_1q^8\end{vmatrix}=a_1^3\begin{vmatrix}1 & q & q^2\\ q^3 & q^4 & q^5\\ q^6 & q^7 & q^8\end{vmatrix}=a_1^3q^9\begin{vmatrix}1 & q & q^2\\ 1 & q & q^2\\ 1 & q & q^2\end{vmatrix}=0$	**3p**

1.)	**Din oficiu**	**1p**
	Dacă $a<0 \Rightarrow \lim\limits_{x\to+\infty}\left(\sqrt{ax^2+x+3}\right)=\sqrt{-\infty}$ nu se defineşte	**1p**
	Dacă $a=0 \Rightarrow \lim\limits_{x\to+\infty}\left(x+1-\sqrt{x+3}\right)=+\infty$	**2p**
	Dacă $a>0 \Rightarrow \lim\limits_{x\to+\infty}\left(x+1-\sqrt{ax^2+x+3}\right)=\lim\limits_{x\to+\infty}\dfrac{(1-a)x^2+x-2}{x+1+\sqrt{ax^2+x+3}}=$ $=\lim\limits_{x\to+\infty}x\dfrac{1-a}{1+\sqrt{a}}=\lim\limits_{x\to+\infty}x\left(1-\sqrt{a}\right)$	**2p**
	Dacă $a\in(0;1) \Rightarrow \lim\limits_{x\to+\infty}\left(x+1-\sqrt{ax^2+x+3}\right)=+\infty$	**1p**
	Dacă $a=1 \Rightarrow \lim\limits_{x\to+\infty}\left(x+1-\sqrt{x^2+x+3}\right)=\lim\limits_{x\to+\infty}\dfrac{x-2}{x+1+\sqrt{x^2+x+3}}=\dfrac{1}{2}$	**2p**
	Dacă $a\in(1;+\infty) \Rightarrow \lim\limits_{x\to+\infty}\left(x+1-\sqrt{ax^2+x+3}\right)=-\infty$	**1p**

3.)	**Din oficiu**	**1p**
a)	$(d): y = -x$ ecuaţia bisectoarei a doua , $C(x_C, y_C) \in d$	**2p**
	Condiţia de coliniaritate: $\begin{vmatrix} 4 & 4 & 1 \\ 10 & 2 & 1 \\ x_C & -x_C & 1 \end{vmatrix} = 0 \Rightarrow -4x_C - 32 = 0 \Rightarrow x_C = -8 \Rightarrow C(-8,8)$	**2p**
b)	$D \in (d):\ x - 2y + 4 = 0 \Rightarrow D(2y_D - 4,\ y_D)$	**1p**
	$A = \frac{1}{2}\lvert\Delta\rvert, \Delta = \begin{vmatrix} 4 & 4 & 1 \\ 10 & 2 & 1 \\ 2y_D - 4 & y_D & 1 \end{vmatrix} = 10y_D - 40$	**2p**
	$\lvert 10y_D - 40\rvert = 10 \Rightarrow y_D = 5$ sau $y_D = 3$, $D_1(6,5)$, $D_2(2,3)$	**2p**