OLIMPIADA DE MATEMATICĂ

ETAPA LOCALĂ

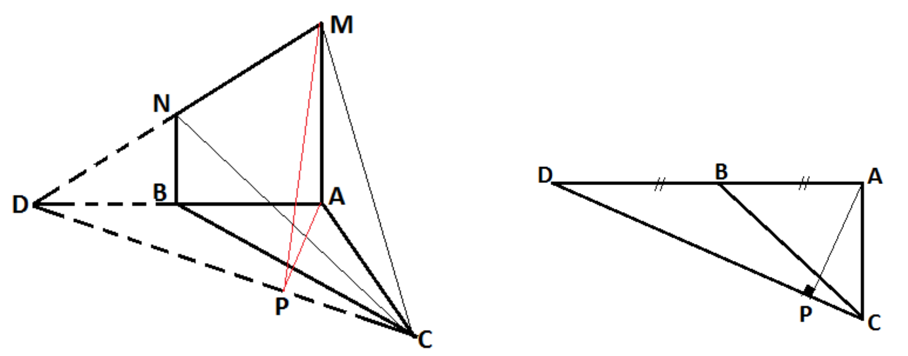
28 februarie 2015

BAREM

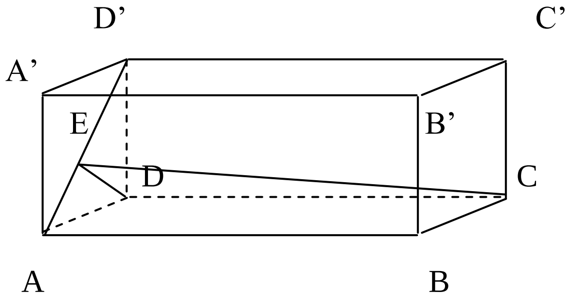
CLASA A VIII-A

1.)	**Din oficiu**	**1p**
a)	$\sqrt{6+4\sqrt{2}}=\sqrt{\left(2+\sqrt{2}\right)^2}=2+\sqrt{2}$	**2p**
	a = 2, b = 1	**1p**
b)	$A=\sqrt{2^{2014}+2^{1008}+1}=\sqrt{\left(2^{1007}\right)^2+2\cdot 2^{1007}\cdot 1+1^2}=\sqrt{\left(2^{1007}+1\right)^2}=2^{1007}+1\in\mathbb{N}$	**2p**
	$B=\sqrt{2^{2016}-2^{1010}+2^{1009}+1}=\sqrt{\left(2^{1008}\right)^2-2^{1009}\cdot\left(2-1\right)+1}=$ $=\sqrt{\left(2^{1008}\right)^2-2\cdot 2^{1008}\cdot 1+1^2}=\sqrt{\left(2^{1008}-1\right)^2}=2^{1008}-1\in\mathbb{N}$	**2p**
	Numărul numerelor naturale din intervalul $(A;B)=\left(2^{1007}+1;\,2^{1008}-1\right)$ este $2^{1008}-1-\left(2^{1007}+1\right)-1=2^{1008}-2^{1007}-3=2^{1007}\cdot\left(2-1\right)-3=2^{1007}-3$	**2p**

2.)	**Din oficiu**	**1p**
	Egalitatea se scrie: $a^2b+a^2c+2abc-2a^3-b^2c-bc^2=0$	**1p**
	$a^2\left(b+c\right)-2a\left(a^2-bc\right)-bc\left(b+c\right)=0$	**2p**
	$\left(b+c\right)\left(a^2-bc\right)-2a\left(a^2-bc\right)=0$	**2p**
	$\left(a^2-bc\right)\left(b+c-2a\right)=0$	**1p**
	$a^2-bc=0$ sau $b+c-2a=0$	**1p**
	$a^2=bc$ sau $b+c=2a$	**1p**
	Deci *a* este media geometrică sau media aritmetică a numerelor *b* și *c*.	**1p**

3.)	**Din oficiu**	**1p**
		**1p**
a)	Prin calcul direct se determină $BC=\sqrt{28}$ cm, $MC=\sqrt{20}=2\sqrt{5}$ cm, $NC=\sqrt{29}$ cm și $MN=\sqrt{13}$ cm.	**2p**
	$\Delta MNC:\sqrt{13}<\sqrt{20}<\sqrt{29}$, $\sqrt{29}^2\neq\sqrt{13}^2+\sqrt{20}^2\overset{recTP}{\Rightarrow}$ nu este dreptunghic	**1p**

b)	determinarea dreptei de intersecție: $MN \cap AB = \{D\}$, $(ABC) \cap (MNC) = DC$	**2p**
	$\left.\begin{array}{l} M \notin (ABC),\ DC \subset (ABC) \\ MA \perp (ABC) \\ AP \perp DC \end{array}\right\} \overset{t3\perp}{\Rightarrow} MP \perp DC \Rightarrow MP = d(M, DC)$	**1p**
	$MA \perp AB, NB \perp AB \Rightarrow MA \parallel NB$ și $NB = \frac{MA}{2} \Rightarrow NB$ linie mijlocie în ΔMDA. Deci $DB = BA = 2\sqrt{3}$ cm și $AD = 4\sqrt{3}$ cm.	**1p**
	$\Delta DAC : t.Pit. \Rightarrow DC = 8$ cm, înălțimea $AP = \frac{AD \cdot AC}{DC} \Rightarrow AP = 2\sqrt{3}$ cm $\Delta MAP : t.Pit. \Rightarrow MP = 4$ cm $= d(M, DC)$	**1p**

4.)	**Din oficiu**	**1p**
	Desen: 	**1p**
a)	$DA \parallel BC \Rightarrow m(\widehat{AD'; BC}) = m(\widehat{AD'; AD}) = m(D'\hat{A}D)$ $\sin D'\hat{A}D = \frac{D'D}{AD'}$	**2p**
	$D'A^2 = D'D^2 + AD^2 = 3^2 + \left(3\sqrt{2}\right)^2 \Rightarrow D'A = 3\sqrt{3}$ Deci, $\sin D'\hat{A}D = \frac{3\sqrt{2}}{3\sqrt{3}} = \frac{\sqrt{6}}{3}$	**2p**
b)	Fie $DE \perp AD', E \in AD'$ $CD \perp AD; CD \perp D'D; AD, D'D \subset (ADD'); AD \cap D'D = \{D\} \Rightarrow CD \perp (ADD')$ $CD \perp (ADD'), AD' \subset (ADD'), DE \perp AD' \Rightarrow CE \perp AD'$. Deci $d(C; AD') = CE$.	**2p**
	$DE \cdot AD' = DD' \cdot AD \Rightarrow DE = \frac{AD \cdot DD'}{AD'} = \frac{3 \cdot 3\sqrt{2}}{3\sqrt{3}} = \sqrt{6}$ $CD \perp (ADD'), DE \subset (ADD') \Rightarrow CD \perp DE \Rightarrow CE^2 = CD^2 + DE^2 = 6^2 + \sqrt{6}^2 \Rightarrow$ $\Rightarrow CE = \sqrt{42}$. Deci $d(C; AD') = \sqrt{42}$ cm.	**2p**