T1

CONCURSUL NAŢIONAL DE MATEMATICĂ APLICATĂ
"ADOLF HAIMOVICI"
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CLASA A XI-A
Filiera tehnologică: profil tehnic-toate calificările profesionale

BAREM DE CORECTARE

SUBIECTUL I

a) $A(-1)=\begin{pmatrix}0&1&0\\0&0&1\\-1&0&0\end{pmatrix}$ $A(-2)=\begin{pmatrix}0&1&0\\0&0&1\\-2&0&0\end{pmatrix}$ $A(-1)-2\,{}^tA(-2)=\begin{pmatrix}0&1&4\\-2&0&1\\-1&-2&0\end{pmatrix}$	1p 1p
b) $A^2(a)=\begin{pmatrix}0&0&1\\a&0&0\\0&a&0\end{pmatrix}$ $A^3(a)=\begin{pmatrix}a&0&0\\0&a&0\\0&0&a\end{pmatrix}$ $\det(A(a)+A^2(a)+A^3(\text{a}))=\begin{vmatrix}a&1&1\\a&a&1\\a&a&a\end{vmatrix}=$ $=a(a-1)^2$ $\Rightarrow a\in\{0,1\}$	1p 1p
c) $A^3=aI_3, A^6=a^2I_3, A^{2016}=a^{672}I_3$ $I_3+A^3+A^6+...+A^{2016}=(1+a+a^2+...+a^{672})I_3$ $a\neq1\Rightarrow I_3+A^3+A^6+...+A^{2016}=\dfrac{a^{673}-1}{a-1}I_3$	1p 1p 1p

SUBIECTUL II

a) $A_1(2,2), A_2(4,3)$, aplică formula pentru ecuaţia dreptei $A_1A_2:-x+2y-2=0$	1p 1p
b) $A_{\Delta A_nA_{n+1}A_{n+2}}=\frac{1}{2}\lvert\Delta\rvert, \Delta=\begin{vmatrix}2^n&n+1&1\\2^{n+1}&n+2&1\\2^{n+2}&n+3&1\end{vmatrix}$ $\lvert\Delta\rvert=2^n$ $A_{\Delta A_nA_{n+1}A_{n+2}}=2^{n-1}=1024\Rightarrow n=11$	1p 1p 1p
c) $A_0(1,1)\Rightarrow B_0(-1,1), C(1,-1)$, $A_1(2,2)\Rightarrow B_1(-2,2)$ $\begin{vmatrix}-1&1&1\\1&-1&1\\-2&2&1\end{vmatrix}=0\Rightarrow B_0, B_1, C$ coliniare	1p 1p

SUBIECTUL III

a) Pentru $\forall x \in \mathbf{R} \setminus \{1,3\}$ f are limită. $l_d(3)=1, l_s(3)=(3+a)^2 \Rightarrow a \in \{-4,-2\}$ $l_d(1)=(1+a)^2+4, l_s(1)=b \Rightarrow b \in \{5,13\}$ Concluzie $(a,b) \in \{(-4,13),(-2,5)\}$	1p 1p 1p
b) $\lim\limits_{x\to\infty} f(x)=\infty, \lim\limits_{x\to-\infty} f(x)=4$, f admite asimptota orizontală la -∞ $y=4$ $l_d(3)=1$ $l_s(1)=5$ $\Rightarrow f$ nu are asimptote verticale $\lim\limits_{x\to\infty}\frac{f(x)}{x}=1, \lim\limits_{x\to\infty}(f(x)-x)=-3 \Rightarrow y=x-3$ asimptotă oblică la ∞	1p 1p 1p 1p

SUBIECTUL IV

1.a) $\lim\limits_{x\to5}\frac{tg(5-x)}{x^2-6x+5}=\lim\limits_{x\to5}\frac{tg(5-x)}{(5-x)}\cdot\frac{5-x}{(x-1)(x-5)}=$ $=\lim\limits_{x\to5}\frac{-1}{x-1}=\frac{-1}{4}$	1p 1p
1.b) $\lim\limits_{x\to-\infty}\frac{\sqrt{x^2-1}+2016x}{-2015x+2}=\lim\limits_{x\to-\infty}\frac{\lvert x\rvert\sqrt{1-\frac{1}{x^2}}+2016x}{-2015x+2}$ $=\lim\limits_{x\to-\infty}\frac{x\left(-\sqrt{1-\frac{1}{x^2}}+2016\right)}{x\left(-2015+\frac{2}{x}\right)}=$ $=\frac{2015}{-2015}=-1$	1p 1p
2.Pentru $a\neq9 \Rightarrow \lim\limits_{x\to2}\frac{3^x-a}{x^2-4}=\frac{9-a}{\pm0}=\pm\infty$, nu convine Pentru $a=9 \Rightarrow \lim\limits_{x\to2}\frac{3^x-9}{x^2-4}=\lim\limits_{x\to2}\frac{3^2(3^{x-2}-1)}{(x-2)(x+2)}=\lim\limits_{x\to2}\frac{9\ln3}{x+2}=\frac{9\ln3}{4}$	1p 2p