T3

CONCURSUL NAŢIONAL DE MATEMATICĂ APLICATĂ
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CLASA A XI-A
Filiera teoretică: profil real, specializarea ştiinţe ale naturii

BAREM DE CORECTARE ŞI NOTARE

SUBIECTUL I

a) $A^2(a)=\begin{pmatrix}0&0&1\\a&0&0\\0&a&0\end{pmatrix}$ $A^3(a)=\begin{pmatrix}a&0&0\\0&a&0\\0&0&a\end{pmatrix}$ $\det(A(a)+A^2(a)+A^3(a))=\begin{vmatrix}a&1&1\\a&a&1\\a&a&a\end{vmatrix}=$ $=a(a-1)^2$ $\Rightarrow a\in\{0,1\}$	1p 1p
b) $A^3=aI_3, A^6=a^2I_3, A^{2016}=a^{672}I_3$ $I_3+A^3+A^6+...+A^{2016}=(1+a+a^2+...+a^{672})I_3$; $a\neq 1\Rightarrow I_3+A^3+A^6+...+A^{2016}=\dfrac{a^{673}-1}{a-1}I_3$	1p 1p
c) $\sum_{k=1}^{9}A(a^k+a^{3k})=\sum_{k=1}^{9}\begin{pmatrix}0&1&0\\0&0&1\\a^k+a^{3k}&0&0\end{pmatrix}=\begin{pmatrix}0&9&0\\0&0&9\\\sum_{k=1}^{9}\left(a^k+a^{3k}\right)&0&0\end{pmatrix}$ $a\in\mathbf{C}\setminus\mathbf{R}, a^2+a+1=0\Rightarrow a^3=1$ $\sum_{k=1}^{9}\left(a^k+a^{3k}\right)=\sum_{k=1}^{9}a^k+\sum_{k=1}^{9}a^{3k}=\dfrac{a(a^9-1)}{a-1}+9=9$	1p 1p 1p

SUBIECTUL II

a) $A_1(2,2), A_2(4,3)$, aplică formula pentru ecuaţia dreptei $A_1A_2: -x+2y-2=0$	1p 1p
b) $A_{\Delta A_nA_{n+1}A_{n+2}}=\frac{1}{2}\lvert\Delta\rvert, \Delta=\begin{vmatrix}2^n&n+1&1\\2^{n+1}&n+2&1\\2^{n+2}&n+3&1\end{vmatrix}$ $\lvert\Delta\rvert=2^n$ $A_{\Delta A_nA_{n+1}A_{n+2}}=2^{n-1}=1024\Rightarrow n=11$	1p 1p 1p
c) $A_0(1,1)\Rightarrow B_0(-1,1), C(1,-1)$, $A_1(2,2)\Rightarrow B_1(-2,2)$ $\begin{vmatrix}-1&1&1\\1&-1&1\\-2&2&1\end{vmatrix}=0\Rightarrow B_0,B_1,C$ coliniare	1p 1p

SUBIECTUL III

a) Pentru $\forall x \in \mathbf{R} \setminus \{1,3\}$ f are limită. $l_d(3)=1, l_s(3)=(3+a)^2+(3-b)^2$ $l_d(1)=(1+a)^2+(1-b)^2, l_s(1)=5$ $\begin{cases}(3+a)^2+(3-b)^2=1\\(1+a)^2+(1-b)^2=5\end{cases} \Rightarrow a=-3, b=2 \text{ sau } a=-2, b=3$	1p 1p 2p
b) $\lim\limits_{x\to\infty} f(x)=0, \lim\limits_{x\to-\infty} f(x)=4$, f admite asimptota orizontală $y=0$ la ∞ şi asimptotă orizontală $y=4$ la -∞, deci f nu admite asimptote oblice $l_d(3)=1$ $l_s(1)=5$ $\Rightarrow f$ nu are asimptote vertical .	1p 1p 1p

SUBIECTUL IV

a)pentru $a<0 \Rightarrow \lim\limits_{x\to\infty} f(x)=\infty$, nu convine pentru $a>0 \Rightarrow \lim\limits_{x\to\infty}\left(\sqrt{9x^2+bx}-ax\right)=\lim\limits_{x\to\infty}\frac{(9-a^2)x^2+bx}{\sqrt{9x^2+bx}+ax}=\lim\limits_{x\to\infty}\frac{(9-a^2)x^2+bx}{x\left(\sqrt{9+\frac{b}{x}}+x\right)}=2015$ $\begin{cases}9-a^2=0\\\frac{b}{3+a}=2015\end{cases} \Rightarrow a=3, b=12090$	1p 3p 1p
b) $\lim\limits_{x\to0}\frac{f(x)}{x}=\lim\limits_{x\to0}\frac{\sqrt{9x^2}}{x}=\lim\limits_{x\to0}\frac{3\|x\|}{x}$ $\Rightarrow l_s(0)=-3, l_d(0)=3 \Rightarrow$ f nu are limită în 0 .	1p 1p