

## Concursul Național de Matematică Aplicată „Adolf Haimovici”

## Etapa Locală

Maramureș – 8 februarie 2025

Clasa a XI - a

Secțiunea H2

Filiera teoretică, profil real, specializarea științe ale naturii

## Barem de corectare și notare

**1. a)**  $\det(A) = a^2 - 3b^2 = 1$

Dacă  $b = 1, a = \pm 2$ , dacă  $b = 0, a = \pm 1$  .....1pPosibile exemple  $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$  ..... 2p

**b)**  $A \cdot B = \begin{pmatrix} a & 3b \\ b & a \end{pmatrix} \cdot \begin{pmatrix} c & 3d \\ d & c \end{pmatrix} = \begin{pmatrix} ac + 3bd & 3(ad + bc) \\ ad + bc & ac + 3bd \end{pmatrix} \in M_2(\mathbb{Z}),$

deci  $A \cdot B \in M$  .....2p

**c)** Fie  $A = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} \in M, \det(A) = 1,$

conform **b)**,  $A^n \in M$  și  $\det(A^n) = 1, \forall n \in \mathbb{N}^*$  .....1pDeoarece  $A$  are elementele strict pozitive,  $A^{n-m} \neq I_2, \forall n > m; m, n \in \mathbb{N}^*$ ;Prin urmare  $A^n \neq A^m, \forall m \neq n$ . Rezultă  $A, A^2, \dots, A^{2025} \in M$  ..... 1p

**2. a)** Fie  $S = A_1 + A_2 + A_3 + \dots + A_{99}$

$$S = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{pmatrix} + \begin{pmatrix} 2 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{pmatrix} + \dots \quad \dots 1p$$

$$S_{11} = 3 \cdot 1 + 3 \cdot 2 + \dots + 3 \cdot 33 = 1683$$

$$S_{12} = S_{11}$$

$$S_{13} = 3 \cdot 2 + \dots + 3 \cdot 34 = 1782$$

$$S_{21} = (1 + 2 + 2) + (2 + 3 + 3) + \dots + (33 + 34 + 34) =$$

$$= \frac{33 \cdot 34}{2} + 2 \cdot \left( \frac{34 \cdot 35}{2} - 1 \right) = 1749$$

$$S_{22} = (0 + 1 + 1) + (1 + 2 + 2) + (2 + 3 + 3) + \dots + (32 + 33 + 33) =$$

$$= \frac{33 \cdot 32}{2} + 2 \cdot \frac{34 \cdot 33}{2} = 1650$$

$$S_{23} = S_{22}$$

$$S_{31} = (0 + 0 + 1) + (1 + 1 + 2) + \dots + (32 + 32 + 33) = 32 \cdot 33 + \frac{33 \cdot 34}{2} = 1617$$

$$S_{32} = (1 + 1 + 2) + \dots + (33 + 33 + 34) = 34 \cdot 33 + \frac{35 \cdot 34}{2} - 1 = 1716$$

$$S_{33} = S_{31}$$

$$S = \begin{pmatrix} 1683 & 1683 & 1782 \\ 1749 & 1650 & 1650 \\ 1617 & 1716 & 1617 \end{pmatrix} \quad \dots 3p$$

b) Fie  $X = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$

$$AX = XA \Leftrightarrow \begin{pmatrix} g & h & i \\ a & b & c \\ d & e & f \end{pmatrix} = \begin{pmatrix} b & c & a \\ e & f & d \\ h & i & g \end{pmatrix} \Rightarrow X = \begin{pmatrix} a & b & c \\ c & a & b \\ b & c & a \end{pmatrix} \in M \quad \dots\dots\dots 3p$$

3. a)  $\lim_{x \rightarrow -\infty} (\sqrt{4x^2 + 8x - 5} + 2x) = \lim_{x \rightarrow -\infty} \frac{4x^2 + 8x - 5 - 4x^2}{\sqrt{4x^2 + 8x - 5} - 2x} = -2 \quad \dots\dots\dots 3p$

b)  $-1 = \lim_{x \rightarrow -\infty} (ax + \sqrt{bx^2 + cx - 1}) = \lim_{x \rightarrow -\infty} \frac{a^2x^2 - (bx^2 + cx - 1)}{ax - \sqrt{bx^2 + cx - 1}} \Rightarrow a^2 = b \dots\dots\dots 1p$

$$-1 = \lim_{x \rightarrow -\infty} \frac{-cx + 1}{ax - \sqrt{a^2x^2 + cx - 1}} = \lim_{x \rightarrow -\infty} \frac{-cx + 1}{ax \left( 1 + \sqrt{1 + \frac{c}{x} - \frac{1}{x^2}} \right)} \Rightarrow \frac{-c}{2a} = -1 \Rightarrow$$

$$c = 2a \dots\dots\dots 1p$$

$$\Rightarrow f(x) = ax + \sqrt{a^2x^2 + 2ax - 1}$$

$$2 = \lim_{x \rightarrow \infty} \frac{f(x)}{x} = \lim_{x \rightarrow \infty} \left( a + \sqrt{a^2 + \frac{2a}{x} - \frac{1}{x^2}} \right) = 2a \Rightarrow a = 1 > 0 \quad \dots\dots\dots 1p$$

$$b = 1 > 0 \text{ și } c = 2 \in \mathbb{R}$$

$$\text{Avem } a = 1, b = 1, c = 2 \dots\dots\dots 1p$$

4.

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin(x) + \sin(3x) + \sin(5x) + \dots + \sin(2025x)}{x} \\ = \lim_{x \rightarrow 0} \left( \frac{\sin x}{x} + \frac{\sin 3x}{x} + \dots + \frac{\sin 2025x}{x} \right) = \dots \quad 2p \\ = 1 + 3 + 5 + \dots + 2025 = 1013^2 \quad \dots\dots\dots 5p \end{aligned}$$