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OLIMPIADA NAŢIONALĂ DE MATEMATICĂ

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Clasa a X-a

Barem de corectare

1. a) $7+5\sqrt{2}=(1+\sqrt{2})^3$ şi $7-5\sqrt{2}=(1-\sqrt{2})^3$ ……………………………..……3p

$\sqrt[3]{7+5\sqrt{2}}$ - $\sqrt[3]{7-5\sqrt{2}} = 1+\sqrt{2}-1+\sqrt{2}=2\sqrt{2}\in \mathbf{R}\backslash\mathbf{Q}$ ……........……….……..……2p

b) $(n+1)^{\frac{1}{n}} > (n+2)^{\frac{1}{n+1}} \Leftrightarrow \left(\frac{n+1}{n+2}\right)^n > \frac{1}{n+1}$ 1p

Ineg. lui Bernoulli $(1+x)^n \geq 1+nx$, $\forall\, n\in \mathbf{N}$, $x > -1$ pentru $x=-\frac{1}{n+2}$

$\Rightarrow \left(\frac{n+1}{n+2}\right)^n \geq \frac{2}{n+2} > \frac{1}{n+1}$ 1p

2. a) $\left(\frac{3a+4b}{3}\right)^2 = 6ab \Leftrightarrow 9a^2-30ab+16\,b^2=0$ 3p

$a=\frac{2b}{3}$ sau $a=\frac{8b}{3}$ 1p

$a > b > 0 \Rightarrow \frac{a}{b} > 1 \Rightarrow \frac{a}{b}=\frac{8}{3}$ 1p

b) $a=\log_7 98 = 2+\log_7 2 = 2+x$, unde $x=\log_7 2$ 1p

$b=\log_2 28 = 2+\frac{1}{x} \Rightarrow \frac{1}{a-1}+\frac{1}{b-1}=\frac{1}{x+1}+\frac{x}{x+1}=1$..1p

3. a) $f(2-i)=10-i$ 3p

$|f(2-i)|=\sqrt{101}$ 2p

b) Ecuaţia $f(z)=2\bar{z}+3z=y$, $y\in \mathbf{C}$ are soluţie unică $z=\frac{3y-2\bar{y}}{5}$ 1p

$\Rightarrow f$ este bijectivă $\Rightarrow f$ este inversabilă $\Rightarrow f^{-1}:\mathbf{C}\to\mathbf{C}$, $f(y)^{-1}=\frac{3y-2\bar{y}}{5}$ 1p

4. a) $\varepsilon^3=1$ si $\varepsilon^2+\varepsilon=-1$ 2p

$(x+\varepsilon y+\varepsilon^2 z)(x+\varepsilon^2 y+\varepsilon z) = x^2+y^2+z^2+(xy+yz+zx)(\varepsilon^2+\varepsilon)=$

$= x^2+y^2+z^2-xy-yz-zx$ 2p

b) $\frac{c-a}{b-a}=\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}=\frac{1}{\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}}=-\frac{1}{\varepsilon}$ 1p

$\Rightarrow a\varepsilon^2+b+c\varepsilon=0 \Rightarrow a+b\varepsilon+c\varepsilon^2=0$ 1p

Conform a) $\Rightarrow a^2+b^2+c^2-ab-bc-ca=0 \Rightarrow a^2+b^2+c^2=ab+bc+ca$ 1p