OLIMPIADA DE MATEMATICĂ
ETAPA LOCALĂ
14 DECEMBRIE 2019
BAREM

CLASA a VIII-a

Subiectul 1.		
a)	$\frac{1}{\sqrt{2}-10}-\frac{1}{\sqrt{2}-9}=\frac{\left(\sqrt{2}-9\right)-\left(\sqrt{2}-10\right)}{\left(\sqrt{2}-10\right)\left(\sqrt{2}-9\right)}=\frac{1}{\left(\sqrt{2}-10\right)\left(\sqrt{2}-9\right)}$	1p.
	$\frac{1}{\left\|\left(\sqrt{2}-9\right)\left(\sqrt{2}-10\right)\right\|}=\frac{1}{\left\|\sqrt{2}-9\right\|\left\|\sqrt{2}-10\right\|}=\frac{1}{\left(9-\sqrt{2}\right)\left(10-\sqrt{2}\right)}=\frac{1}{\left(\sqrt{2}-10\right)\left(\sqrt{2}-9\right)}$	1p
b)	$\left.\begin{array}{l}\frac{1}{\sqrt{\left(\sqrt{2}-2\right)^2}\cdot\sqrt{\left(\sqrt{2}-3\right)^2}}=\frac{1}{\left(\sqrt{2}-3\right)}-\frac{1}{\left(\sqrt{2}-2\right)}\\ \frac{1}{\sqrt{\left(\sqrt{2}-3\right)^2}\cdot\sqrt{\left(\sqrt{2}-4\right)^2}}=\frac{1}{\left(\sqrt{2}-4\right)}-\frac{1}{\left(\sqrt{2}-3\right)}\\ \frac{1}{\sqrt{\left(\sqrt{2}-9\right)^2}\cdot\sqrt{\left(\sqrt{2}-10\right)^2}}=\frac{1}{\left(\sqrt{2}-10\right)}-\frac{1}{\left(\sqrt{2}-9\right)}\end{array}\right\}\Rightarrow$	3p
	$y=\frac{1}{\left(\sqrt{2}-10\right)}-\frac{1}{\left(\sqrt{2}-2\right)}=\frac{8}{\left(\sqrt{2}-10\right)\cdot\left(\sqrt{2}-2\right)}=\frac{8}{22-12\sqrt{2}}=\frac{4}{11-6\sqrt{2}}$	1p
	$\frac{y}{x}=\frac{4}{11-6\sqrt{2}}:\frac{44+24\sqrt{2}}{98}=\frac{4}{11-6\sqrt{2}}\cdot\frac{98}{4\left(11+6\sqrt{2}\right)}=\frac{98}{49}=2\in\mathbb{Z}$	1p
Subiectul 2.		
a)	$n=\left(k^2+k+1\right)^2$	3p
b)	$\sqrt{1+\frac{1}{k^2}+\frac{1}{(k+1)^2}}=\sqrt{\frac{(k^2+k+1)^2}{k^2(k+1)^2}}=\frac{k^2+k+1}{k(k+1)}=1+\frac{1}{k}-\frac{1}{k+1}$ $a=1+\frac{1}{1}-\frac{1}{2}+1+\frac{1}{2}-\frac{1}{3}+\ldots+1+\frac{1}{k}-\frac{1}{k+1}$ $a=k+1-\frac{1}{k+1}\Leftrightarrow a=k+\frac{k}{k+1}$ Cum $0<\frac{k}{k+1}<1$, obţinem $\left[a\right]=k$, deci $k=1000$	4p

Subiectul 3.		
a)	Demonstrarea faptului că $A'M \perp AM$	1p
	Demonstrarea faptului că $A'M \perp (ABM)$ şi $d(A',(ABM)) = 6\sqrt{2}$ cm	2p
b)	$D'N \parallel MC$	1p
	$\sphericalangle(AM;D'N) = \sphericalangle(AM;MC) = \sphericalangle AMC$	1p
	$AC = MC = 10$ cm, $AM = 6\sqrt{2}$ cm	1p
	$\text{tg}(\sphericalangle AMC) = \frac{\sqrt{41}}{3}$	1p
Subiectul 4.		
a)	Notăm cu x lungimea muchiilor laterale $AB = AC = BC = x\sqrt{2},\ VO = \frac{x\sqrt{3}}{3},\ SO = \frac{2x\sqrt{3}}{3}$ $SA = SB = x\sqrt{2}$ (teorema lui Pitagora $\Delta SOB; \Delta SOA$) $\frac{P_{SAB}}{P_{VAB}} = \frac{3x\sqrt{2}}{2x + x\sqrt{2}} = \frac{3\sqrt{2}}{2+\sqrt{2}} = 3\sqrt{2} - 3$	2p
b)	Demonstrarea faptului că $SA \parallel VM$ şi concluzia	2p
c)	Demonstrarea faptului că $SA \perp VA$ (reciproca TP sau altă metodă) şi că $VA \parallel QC$ (VAQC paralelogram), deci $SA \perp CQ$ Demonstrarea faptului că $SA \perp BC$ şi concluzia	3p