BAREM DE EVALUARE ŞI DE NOTARE

Varianta 23

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SUBIECTUL I (30 de puncte)

1.	$\Delta = 169$ $x_1 = \frac{1}{7}$; $x_2 = 2$ $x \in [\frac{1}{7}, 2]$	**1p** **2p** **2p**
2.	**N** = Numărul submulţimilor cu 3 elemente ale mulţimii A care conţine elementul 5 este egal cu numărul submulţimilor cu 2 elemente ale mulţimii A\{5}; **N** = $C_9^2 = 36$	**3p** **2p**
3.	Nr.caz.fav. = 81; Nr.caz.posib.= 90; $p = \frac{\text{nr.caz.fav.}}{\text{nr.caz.posib.}} = 0,9$	**2p** **2p** **1p**
4.	$\begin{cases} x-1>0 \\ 6x-5>0 \end{cases} \Rightarrow x \in (1,+\infty)$; $6x^2 - 11x - 95 = 0$; $\Delta = 2401$; $x_1 = 5 \in (1,+\infty)$; $x_2 = \frac{-19}{6} \notin (1,+\infty)$	**1p** **1p** **1p** **1p** **1p**
5.	$d^2(A,B) = (m+5)^2 + (-m-7)^2 = 100$; $m^2 + 12m - 13 = 0$; $\Delta = 196$; $m_1 = -13$; $m_2 = 1$	**2p** **2p** **1p**
6.	$\vec{u} + \vec{v} = 6\vec{i} - 3\vec{j}$; $\lvert\vec{u} + \vec{v}\rvert = 3\sqrt{5}$	**2p** **3p**

SUBIECTUL al II-lea (30 de puncte)

1. **a)**	$\text{rang}A \geq 2 \Leftrightarrow x \in \mathbb{R} \setminus \{1\}$; $\text{rang}A \geq 2 \Leftrightarrow \det A \neq 0$; $\text{rang}A = 2 \Leftrightarrow x = -2$	**1p** **2p** **2p**
b)	Pentru $x = -2 \Rightarrow A^* = \begin{pmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{pmatrix}$; $\det A^* = 0$	**3p** **2p**
c)	$Y \in M_{1,3}(\mathbb{R}) \Leftrightarrow Y = \begin{pmatrix} x & y & z \end{pmatrix}; x,y,z \in \mathbb{R}$; $x = -1$ şi $YA = B \Rightarrow x = y = z = 1$, $Y = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$	**2p** **3p**
2.	$f = x^3 - 9x^2 - x + 9 = (x^2 - 1)(x - 9)$;	**3p**

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a)	$q = x - 9;\ r = 0$	**2p**
b)	x_1, x_2, x_3 rădăcini $\Rightarrow f(x_1) = f(x_2) = f(x_3) = 0$ şi $x_1 + x_2 + x_3 = 9$; $x_1^3 + x_2^3 + x_3^3 = 9(x_1^2 + x_2^2 + x_3^2) + (x_1 + x_2 + x_3) - 27 = 9(x_1^2 + x_2^2 + x_3^2) - 18$	**2p** **3p**
c)	$f(3^x) = 0 \Rightarrow (3^x - 1)(3^x + 1)(3^x - 9) = 0$; $3^x - 1 = 0 \Rightarrow x = 0$; $3^x + 1 = 0 \Rightarrow$ ecuaţie imposibilă; $3^x - 9 = 0 \Rightarrow x = 2$	**2p** **1p** **1p** **1p**

SUBIECTUL al III-lea (30 de puncte)

1. **a)**	$\lim\limits_{x\to\pm\infty} \sqrt[3]{x^3 + 3x - 4} = \pm\infty \Rightarrow G_f$ nu are AO	**1p**
	$\lim\limits_{x\to\pm\infty} \frac{f(x)}{x} = 1$;	**1p**
	$\lim\limits_{x\to\pm\infty} [f(x) - x] = 1$;	**1p**
	$y = x + 1$, asimptotă oblică;	**1p**
	f cotinuă pe **R** $\Rightarrow G_f$ nu are AV	**1p**
b)	$x^3 + 3x^2 - 4 = 0 \Rightarrow x = 1, x = -2$;	**1p**
	$f'(x) = \frac{3x^2 + 6x}{3\sqrt[3]{(x^3 + 3x^2 - 4)^2}} = \frac{x^2 + 2x}{\sqrt[3]{(x^3 + 3x^2 - 4)^2}},\ \forall x \in \mathbb{R} \setminus \{-2, 1\}$;	**2p**
	$f^2(x) \cdot f'(x) = x^2 + 2x,\ \forall x \in \mathbb{R} \setminus \{-2, 1\}$	**2p**
c)	$f(-2) = 0 \Rightarrow$ f nu este derivabilă în $x_0 = -2$;	**1p**
	$d_s = \lim\limits_{x\nearrow -2} \frac{\sqrt[3]{x^3 + 3x^2 - 4}}{x + 2} = \lim\limits_{x\nearrow -2} \sqrt[3]{\frac{x - 1}{x + 2}} = +\infty$;	**2p**
	$d_d = \lim\limits_{x\searrow -2} \frac{\sqrt[3]{x^3 + 3x^2 - 4}}{x + 2} = \lim\limits_{x\searrow -2} \sqrt[3]{\frac{x - 1}{x + 2}} = -\infty$	**2p**
2. **a)**	$f'(x) = 3(x^2 - 1); f'(x) = 0 \Rightarrow x = \pm 1$;	**1p**
	f este strict cresc. $\Leftrightarrow f'(x) > 0 \Leftrightarrow x \in (-\infty, -1)$, respectiv $(1, +\infty)$;	**2p**
	f strict descrescătoare pe $(-1, 1)$; $f(-1) = 4$ maxim local, $f(1) = 0$ minim local	**2p**
b)	$I = \int_2^3 \frac{f(x)}{x - 1} dx = \int_2^3 (x^2 + x - 2) dx$;	**2p**
	$I = \frac{x^3}{3} + \frac{x^2}{2} - 2x \Big\|_2^3 = \frac{41}{6}$	**3p**
c)	$\frac{x^2 - 13}{f(x)} = \frac{2}{x - 1} - \frac{4}{(x - 1)^2} - \frac{1}{x + 2}, \forall x \in [-1, 0]$;	**2p**
	$\mathbf{I} = \int_{-1}^0 \frac{x^2 - 13}{f(x)} dx = 2\ln\|x - 1\| + \frac{4}{x - 1} - \ln\|x + 2\| \Big|_{-1}^0$;	**2p**
	$\mathbf{I} = -2 - 3\ln 2$	**1p**

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