# BAREM DE EVALUARE ŞI DE NOTARE

## Varianta 25

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**SUBIECTUL I (30 de puncte)**

| | | |
|---|---|---|
| **1.** | $(1+i)^4 = -4$ ;<br>$(1+i)^4 = -4$ ;<br>$(1+i)^{2012} - (1-i)^{2012} = (-4)^{503} - (-4)^{503} = 0$ | **1p**<br>**1p**<br>**3p** |
| **2.** | Notăm $3^x = y \Rightarrow 3y^2 - 10y + 3 = 0 \Rightarrow y_1 = 3;\ y_2 = 1/3;$<br>$3^x = 3 \Rightarrow x = 1;$<br>$3^x = 1/3 \Rightarrow x = -1$ | **3p**<br>**1p**<br>**1p** |
| **3.** | $a_6 = a_3 + 3r;$<br>$a_{16} = a_{19} - 3r;$<br>$a_3 + a_{19} = a_6 + a_{16} = 2012$ | **1p**<br>**1p**<br>**3p** |
| **4.** | $C_{n+1}^1 + C_{n+1}^2 = 36 \Rightarrow (n+1)(n+2) = 72;$<br>$n + 1 = 8 \Rightarrow n = 7$ | **3p**<br>**2p** |
| **5.** | Fie M mijlocul segmentului [AB] $\Rightarrow$ M(-1,1);<br>$m_{AB} = -3/4 \Rightarrow m' = 4/3$<br>Ecuaţia mediatoarei lui [AB]: $4x - 3y + 7 = 0$ | **2p**<br>**1p**<br>**2p** |
| **6.** | $\sin^2 x - \cos^2 x = \cos x \Leftrightarrow 2\cos^2 x + \cos x - 1 = 0$ ;<br>$\cos x = -1 \Rightarrow x \in \{\pi + 2k\pi, k \in Z\}$ ;<br>$\cos x = \frac{1}{2} \Rightarrow x \in \{\pm\frac{\pi}{3} + 2k\pi, k \in Z\}$ ;<br>$x \in [0, 2\pi] \Rightarrow x \in \left\{\frac{\pi}{3},\ \pi,\ \frac{5\pi}{3}\right\}$ | **1p**<br>**1p**<br>**2p**<br>**1p** |

**SUBIECTUL al II-lea (30 de puncte)**

| | | |
|---|---|---|
| **1.**<br>**a)** | $H^*(t) = \begin{pmatrix} t & -t\ln t & 0 \\ 0 & t & 0 \\ 0 & 0 & 1 \end{pmatrix}; \forall t > 0$ ;<br>$\det H^*(t) = t^2 \neq 0 \Rightarrow \operatorname{rang} H^*(t) = 3,\ \forall t > 0$ | **3p**<br>**2p** |
| **b)** | $H(x)\cdot H(y) = \begin{pmatrix} 1 & \ln x + \ln y & 0 \\ 0 & 1 & 0 \\ 0 & 0 & xy \end{pmatrix};\ \forall x, y \in (0, +\infty)$ ;<br>Deci $H(x)\cdot H(y) = H(xy);\ \forall x, y \in (0, +\infty)$ | **3p**<br>**2p** |
| **c)** | $H(1)+H(2)+H(3)+\ldots.+H(10) = \begin{pmatrix} 10 & \ln(1\cdot 2\cdot \ldots \cdot 10) & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 55 \end{pmatrix};$<br>$\det[H(1)+H(2)+H(3)+\ldots.+H(10)] = 5500$ | **3p**<br>**2p** |
| **2.**<br>**a)** | $\left.\begin{array}{l} x > 2 \Rightarrow x - 2 > 0 \\ y > 2 \Rightarrow y - 2 > 0 \end{array}\right\} \Rightarrow xy - 2x - 2y + 4 > 0$ ; | **3p** |

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|---|---|---|
| | $x * y = xy - 2x - 2y + 6 \in G;\ \forall x, y \in G;$<br>G este parte stabilă faţă de operaţia * | **1p**<br>**1p** |
| **b)** | Observăm că operaţia * este comutativă;<br>Elementul neutru: e = 3;<br>$x * x' = 3 \Rightarrow x'(x-2) = 2x - 3, \forall x \in G;$<br>$x' = 2 + \frac{1}{x-2} > 0, \forall x \in G$ | **1p**<br>**1p**<br>**1p**<br>**2p** |
| **c)** | $x * y * z = (x-2)(y-2)(z-2) + 2, \forall x, y, z \in G$<br>$\frac{1}{2} * \frac{2}{3} * \cdots * \frac{8}{9} = \frac{-3}{2} \cdot \frac{-4}{3} \cdot \ldots \cdot \frac{-10}{9} + 2 = 7$ | **2p**<br>**3p** |

**SUBIECTUL al III-lea (30 de puncte)**

| | | |
|---|---|---|
| **1.**<br>**a)** | $f'(x) = 2012x^{2011} + 2012, \forall x \in \mathbb{R}$;<br>f(1) = 0;<br>$f'(0) = 2012$;<br>$f(1) + f'(0) = 2012$ | **2p**<br>**1p**<br>**1p**<br>**1p** |
| **b)** | $y - f(1) = f'(1)(x-1)$;<br>y = 4024(x – 1) | **3p**<br>**2p** |
| **c)** | $f''(x) = 2012 \cdot 2011x^{2010}, \forall x \in \mathbb{R}$;<br>$f''(x) \geq 0,\ \forall x \in \mathbb{R} \Rightarrow f$ este convexă pe $\mathbb{R}$ | **3p**<br>**2p** |
| **2.**<br>**a)** | $f(x) = x^3 + 3x,\ \forall x \in \mathbf{R}$ (1)<br>$I = \int_0^1 f(x)dx = \int_0^1 (x^3 + 3x)dx = \frac{x^4}{4} + \frac{3x^2}{2}\Big|_0^1$;<br>$I = \frac{7}{4}$ | **1p**<br>**3p**<br>**1p** |
| **b)** | $f^5(-x) = [(-x)^3 + 3(-x)]^5 = -f^5(x),\ \forall x \in \mathbf{R}$;<br>$f^5$ este funcţie impară $\Rightarrow \int_{-1}^1 f^5(x)dx = 0$ | **3p**<br>**2p** |
| **c)** | $\int_0^x f(t-1)dt = \frac{(t-1)^4}{4} + \frac{3(t-1)^2}{2}\Big|_0^x = \frac{(x-1)^4 + 6(x-1)^2 - 7}{4}$;<br>$\lim_{x\to\infty} \frac{\int_0^x f(t-1)dt}{x^4} = \lim_{x\to\infty} \frac{(x-1)^4 + 6(x-1)^2 - 7}{4x^4} = \frac{1}{4}$ | **3p**<br>**1p** |

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